Should We Tax Capital Income or Wealth?

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August 19, 2026

Latest draft can be found in this link

Abstract

The answer is: we should tax capital income. This conclusion is derived by analysing taxes on capital income and wealth in an infinite-horizon Merton-Samuelson portfolio model where ex ante identical individuals optimally choose consumption and their portfolio of safe and risky assets. Return risk may either be idiosyncratic or aggregate. Both tax reforms and optimal taxes are analysed. Tax reforms raise taxes on capital income and reduce taxes on wealth, such that the net intertemporal price of consumption remains constant. Such tax reforms are found to be weakly welfare improving, because taxes on capital income impose a non-distorting tax on the risk-premium, whereas taxes on wealth do not. Taxing the risk-premium is welfare-improving if risk is idiosyncratic. It is also welfare-improving if risk is aggregate and the government provides state-contingent public goods, as long as the market risk premium exceeds the risk premium on public goods provision. Taxing the risk-premium is welfare-neutral if risk is aggregate with state-contingent lump-sum taxes. For the same distortions, taxes on capital income always generate more revenue than taxes on wealth. Optimal taxes on capital income and wealth are also derived. Optimal taxes on capital income are 100 percent for idiosyncratic risk, between 0 and 100 percent for aggregate risk with state-contingent public goods and indeterminate for aggregate risk with state-contingent lump-sum taxes. Taxes on capital income are used to tax the risk premium, while (negative) taxes on wealth should ensure intertemporal efficiency. Optimal tax policy is therefore equivalent to a tax on capital income joint with an allowance for the safe rate of return or a pure tax on the risk-premium.

1 Introduction

Should capital income or wealth be taxed? Recently, proposals for taxing wealth have greatly gained prominence, see, for example, Piketty (2014) and Saez and Zucman (2019). These proposals go against an historically strong preference, shared by many public-finance economists, that capital income taxes should be preferred over wealth taxes, see, for example, Bastani and Waldenström (2020), Abdel-Kader and De Mooij (2020), Boadway and Pestieau (2023), Cnossen

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and Jacobs (2021) and Hebous et al. (2024).[1] This paper aims to answer the question: should capital income taxes be preferred over wealth taxes?

Despite its obvious relevance, no paper in the public-finance literature has systematically analysed the fundamental differences between taxes on capital income and wealth in the infinite-horizon portfolio-choice model of Merton (1969) and Samuelson (1969), where ex ante identical individuals optimally choose their savings and portfolios of safe bonds and risky equities.[2] Therefore, this paper aims to answer the question whether capital income taxes should be preferred over wealth taxes. Taxes on capital income are economically different from taxes on wealth due to the presence of risk in asset returns. Taxes on wealth are equivalent to taxes on the safe rate of return, while taxes on capital income tax both the safe rate of return and the risk premium. This paper analyses capital taxes in three main cases: i) capital returns have idiosyncratic risks, which are pooled by the government and returned in the form of lower deterministic taxes, ii) capital returns have aggregate risks and the government closes the budget with state-contingent transfers and iii) capital returns have aggregate risks and the government closes the budget with state-contingent public goods. The case with idiosyncratic investment risk corresponds to risks in non-tradable assets, such as shares in proprietorships or closely-held firms, entrepreneurial talent or portfolio-investment ability. The case with aggregate risks corresponds to stock-market risk.

To analyse whether taxes on capital income are better than taxes on wealth, this paper studies both tax reforms and optimal taxes. The tax reforms analyse perturbations of pre-existing tax systems that are not necessarily optimal. The tax reforms increase the tax on capital income and reduce the tax on wealth such that the safe after-tax rate of return remains constant, i.e. the intertemporal price of consumption remains fixed. Such reforms entail a shift of the tax burden on capital away from the safe return towards the risk premium. As a result, the comparative statics of saving and portfolio responses to the tax reform are very sharp. In particular, saving does not respond to such reforms. The portfolio share of risky assets in total wealth generally has a unit elasticity with respect to the tax on capital income due to the Domar-Musgrave effect; individuals adjust their portfolio of risky and safe assets to obtain exactly the same risk exposure as before the tax reform. As expected utility remains constant in the tax reform, the reform points to pure Pareto improvements.[3] Moreover, as a methodological contribution, this paper shows how to derive Roy's identities from the Bellman equation to derive the welfare effects of government instruments.

From the tax-reform analysis follow the main findings of this paper: taxes on capital income

[1] A good case can be made for levying wealth taxes if no capital income taxes can be levied, for example because taxes on capital income are lacking (as in the Netherlands), or capital gains are realisation-based and the very wealthy indefinitely postpone capital-gains realisation (the 'Warren Buffett problem'). See also Saez et al. (2021) and Schelderup and Zoutman (2024).

[2] It is perhaps surprising that nearly half a century has passed since these Nobel-prize-winning contributions and that barely any analysis exists, neither in the public finance nor in the finance literature. The only paper that the author is aware of is Hamilton (1987), which provides a first pass at comparing taxes on wealth and capital income in a portfolio-model with only aggregate risk and very specific assumptions to obtain closed-form solutions. To be sure, the literature has provided important insights from one- and two-period models, see next section with the literature review.

[3] Only if risks are aggregate and the government returns all risky tax revenue in state-contingent lump-sum taxes, the Domar-Musgrave effect is absent. Still, the comparative statics are sharp; the portfolio share does not respond at all.

(weakly) dominate taxes on wealth in welfare terms. Taxes on capital income are welfare-superior if risk in asset returns is idiosyncratic. Intuitively, the government eliminates a market failure by pooling investment risks and becoming a 'silent partner' in equity investments. The government insures investment risk by taxing the risk premium and lowering deterministic taxes. Taxes on capital income are also welfare-superior if risk in asset returns is aggregate and the government provides state-contingent public goods, as long as the risk premium on private goods exceeds the risk premium on public goods. The government then smooths out aggregate risks over state-contingent private and state-contingent public consumption. Taxes on capital income are equally good as taxes on wealth in welfare terms if risk is aggregate and the government can only provide state-contingent transfers. In that case, replacing wealth taxes by capital-income taxes yields no welfare gain (or loss), because the government shares in the investment risk via the tax on capital income – on the one hand – and returns this investment risk via state-contingent transfers – on the other hand. In all cases, the tax on capital income yields more (expected) tax revenue than the tax on wealth for the same intertemporal distortions. Intuitively, the tax base is shifted from a more (wealth) to a less distortionary part of the capital tax base (risk premium). The tax reforms generally encourage risk-taking by the Domar-Musgrave effect, which yields additional tax revenue since the risk-premium is taxed at a higher rate. This effect is absent in the case with aggregate risk and state-contingent lump-sum transfers.

These findings are confirmed by deriving the optimal tax systems for all three cases. In particular, with idiosyncratic risk, the government likes to increase taxes on capital income to 100 percent. By fully appropriating all investment risk, the government fully insures equity risk. Revenues from taxing the risk-premium are rebated via lower wealth taxes to reduce intertemporal distortions. However, a first-best allocation cannot be attained. If the wealth subsidy were to remove all intertemporal distortions, the intertemporal optimality conditions of the government – which follow from the optimal path of public debt – and of the private sector are mutually inconsistent whenever the risk premium is positive. Optimal policy is therefore second best: it features a positive net tax on the safe return to saving and an under-provision of public goods.[4] With aggregate risk and state-contingent lump-sum transfers, the government reaches a first-best allocation. The optimal tax on capital income is indeterminate, and it can lie between 0 and 100 percent, because the government is indifferent between taxing capital income or wealth. For any tax on capital income, a positive wealth subsidy eliminates any intertemporal distortions. With aggregate risk and state-contingent public goods, the optimal tax on capital income is strictly positive, but lower than 100 percent. The optimal tax rate on capital income depends on how effective public goods are in sharing aggregate risks. Also in this case, a positive wealth subsidy eliminates any intertemporal distortions in consumption choices. Consequently, optimal tax policy is equivalent to an allowance for the safe rate-of-return in capital income or a tax on the risk-premium only, while exempting the normal return from

[4]This inconsistency has gone unnoticed in the literature, which does not impose a no-leverage constraint on individual portfolios. Without such a constraint the equity position, and with it the tax base, grows without bound as the tax on capital income approaches 100 percent, so that the optimal-tax problem has no solution. Only when the constraint is imposed the optimal-tax problem is well-defined. However, once the no-leverage constraint binds, the marginal unit of saving earns a return equal to the safe rate plus the risk-premium, which generates the inter-temporal inconsistency. See Section 2.

taxation.[5] The latter is indeed generally optimal in the framework of the current paper, but is generally not supported by the optimal-tax literature, see next section.

This paper is structured as follows. Section 2 discusses the related literature. Section 3 sets up the model. Section 4 analyses the welfare effects of tax reforms that raise the tax on capital income and lower the tax on wealth; it contains the main findings of the paper. Section 5 derives optimal taxes and Section 6 derives optimal time-invariant taxes. Section 7 concludes. The Appendix contains the proofs of all Lemmas and Propositions.

2 Related literature

This paper relates to eight strands in the literature on the taxation of capital income and wealth. The organising principle of this section is methodological: what is the modelling framework in each strand of the literature? The reason for doing so is that the main contribution of this paper – analysing tax reforms and optimal taxes on capital income or wealth inside the dynamic, stochastic Samuelson-Merton model of optimal consumption and portfolio choice – has not been done before.

Taxation and risk-taking: comparative statics. The oldest papers in the literature develop static or two-period portfolio models with return risk to study how proportional taxes on capital income or on wealth affect saving and risk-taking (i.e. the portfolio share of risky assets). Domar and Musgrave (1944) show that a proportional tax on all returns to capital with full loss offsets promotes risk-taking, since the government becomes a silent partner in the investment. Mossin (1968) and Stiglitz (1969) extend this analysis by placing portfolio choice in an expected-utility framework, and derive that the elasticity of the share of risky assets with respect to the net-of-tax rate is minus one when tax changes are compensated (i.e. net of income effects). The share of risky assets declines proportionally with the net-of-tax rate. Stiglitz (1969) also compares various capital taxes. In his model, the wealth tax is levied on end-of-period wealth, and is equivalent to a proportional reduction in initial wealth. It therefore affects risk-taking through an additional wealth effect that is absent under a tax on capital income. Comparing the two at an equal loss in expected utility, the capital income tax induces more risk-taking than the wealth tax. Sandmo (1969), writing at the same time as Stiglitz (1969), analyses optimal portfolio choice alongside optimal intertemporal choice (i.e. saving).[6] Although Sandmo (1969) does not explicitly analyse a wealth tax, he distinguishes between income taxes on all returns and taxes on excess returns only (i.e. the risk premium). An ex ante wealth tax is the complementary case: a tax on the safe (normal) return only. He confirms Mossin (1968) that a tax on the excess return gives a unit-elastic change in the demand for the risky asset, while leaving expected utility, saving and consumption unchanged. All these papers analyse one- or two-period models and mainly derive the comparative statics of given taxes. By analysing a large-horizon consumption–portfolio model, the current paper confirms that the comparative

[5] This efficiency role of the wealth tax weakens if the optimal tax on the safe return to capital should be positive, as many papers in the optimal tax literature have derived – see next section.

[6] Sandmo (1977) further develops the comparative statics of capital-income taxes on risk-taking in a model with an arbitrary number of risky assets.

statics derived in the one- and two-period models extend to large-horizon models. Moreover, it analyses the welfare-economic desirability of taxes on capital income rather than wealth.

Taxation and risk-taking: taxation of the risk premium with idiosyncratic risk. A second strand of the literature studies the taxation of uninsurable idiosyncratic risk. Eaton and Rosen (1980) and Varian (1980) analyse risk in labour earnings and show that a proportional income tax provides social insurance. The government then optimally trades off insurance gains against distortions of taxation. Applied to idiosyncratic return risk, the government pools the idiosyncratic returns of all investors and returns this revenue via lower taxes. Boadway and Spiritus (2025) formalise this argument in a two-period model and show that excess returns should be taxed at 100 percent, while the normal return to saving remains untaxed. This paper partly confirms their findings in the infinite-horizon Merton-Samuelson framework: taxes on capital income are 100 percent. But wealth subsidies or rate-of-return allowances do not eliminate all intertemporal distortions, due to a structural issue that appears only once individual portfolios are subject to a no-leverage constraint. The government prices resources over time at the safe rate of return, while the private sector prices resources at the safe return plus the risk premium if risks are fully insured. Both cannot hold simultaneously in intertemporal equilibrium unless the risk premium is zero. Hence, a first-best allocation cannot be reached, and the finding of Boadway and Spiritus (2025) – that the normal return should remain untaxed if there is only idiosyncratic risk – cannot be sustained. Indeed, optimal policy with idiosyncratic risk is only defined if it is a second-best Ramsey problem in which the government trades off intertemporal distortions against the under-provision of public goods.

The problem in Boadway and Spiritus (2025) is that they do not restrict leverage. The Domar-Musgrave effect implies that individuals respond to a higher tax on excess returns by scaling up their equity position in proportion, so as to leave their after-tax risk exposure – and thereby their expected utility – unchanged. As the tax on capital income approaches 100 percent, the equity position, and with it the tax base, grows without bound. The government therefore collects unbounded revenue from taxing the risk premium at no utility cost to individuals, so that social welfare has no maximum and the optimal-tax problem has no solution. Moreover, at a 100 percent tax individuals are indifferent about the composition of their portfolio, so that the tax base is not determinate. This does not depend on the infinite horizon as the same unboundedness arises with only two periods. Not imposing a no-leverage constraint is not economically innocent. A leveraged position requires a counterparty. With ex ante identical individuals there is none, since aggregate holdings of the safe asset must equal their net supply. Unbounded leverage thus presumes an infinitely elastic supply of safe funds at the exogenous safe rate, which is the partial-equilibrium assumption that the optimal policy solution invalidates. Hence, imposing a no-leverage constraint ensures a well-defined optimal-tax problem, but it also changes optimal tax policy.

Taxation and risk-taking: taxation of the risk premium with aggregate risk. A third strand in the literature studies the welfare implications of taxing risky capital income, where all risk is non-insurable, aggregate risk. Gordon (1985) and Kaplow (1994) develop saving-portfolio models to show that taxing the risk premium raises government revenue, but

this brings no welfare gain since aggregate investment risk is ultimately borne by the same taxpayers, because all risk in tax revenue is ultimately returned to taxpayers in the form of state-contingent taxes.[7] Consequently, with state-contingent rebates of revenue, taxation of the risk premium does not generate a welfare gain. This implies that capital-income taxes and ex ante wealth taxes are equivalent in welfare terms. Related is a largely forgotten paper by Hamilton (1987) who is the only one – to the best knowledge of the author – who has analysed capital taxes in the Merton-Samuelson framework. By making strong assumptions (CRRA utility, discounting stochastic government revenue with a weighted discount rate, no public goods, no debt, no labour income) he shows analytically that both tax instruments are equivalent in welfare terms. This paper confirms the conclusions of this literature in case all risk is aggregate and the government can only use state-contingent lump-sum taxes. This paper generalises Hamilton (1987) to allow for general utility functions and risk processes, exogenous labour income, public debt, public goods and by not taking a short-cut on the government budget constraint. Moreover, this paper also analyses idiosyncratic risks and state-contingent public goods provision and then the equivalence between taxes on capital income and wealth is broken.

Optimal taxation of risky capital income in two-period models. A fourth strand of the literature studies optimal taxes on risky capital income with state-contingent public goods, rather than state-contingent taxes. Christiansen (1993) and Schindler (2008) show that taxing the risk-premium now does generate social value, since it allows the government to smooth aggregate risk over (stochastic) private and public consumption. Boadway and Spiritus (2025) generalise these papers by adding heterogeneity in labour earnings and capital returns and an optimal non-linear tax on labour income. Like Mossin (1968) and Sandmo (1969), all these papers confirm the unit elastic response of risk-taking with respect to taxes on the excess return. Moreover, these papers do not directly compare taxes on capital income and wealth. However, Schindler (2008) and Boadway and Spiritus (2025) in the case where there is no return heterogeneity find that the normal return should typically remain untaxed, while the risk-premium should be taxed.[8] This finding implies that taxes on capital income are welfare superior over taxes on wealth, since the latter only tax normal returns, while taxes on capital income tax all returns, including the risk premium. All papers remain within a two-period structure and do not analyse a fully dynamic-stochastic Merton-Samuelson model. This paper shows that most insights of these papers carry over to the multiple-period framework of Merton-Samuelson.

Dynamic optimal taxation with heterogeneous returns and entrepreneurship. A fifth strand of the literature analyses optimal taxation in two- or multi-period models, but focuses on return heterogeneity caused by capital market frictions or entrepreneurial investment decisions. Guvenen et al. (2023) develop a quantitative macro model where capital returns are heterogeneous due to entrepreneurs facing borrowing constraints. They show that switching

[7] Bulow and Summers (1984) is also related.

[8] Both excess and normal returns should be taxed in Boadway and Spiritus (2025) if capital returns are heterogeneous and contain type-dependent rents as shown in Gerritsen et al. (2025).

from taxes on capital income to wealth raises efficiency by reallocating capital towards more productive and more constrained entrepreneurs. While inequality rises at the same time, the reallocation of capital alleviates borrowing constraints for high-return entrepreneurs, which dominates the welfare loss from higher inequality.[9] Boar and Midrigan (2023) show that this result is fragile and they reach the opposite conclusion to Guvenen et al. (2023). A switch from taxing wealth to taxing capital income makes the allocation of capital indeed less efficient, because productive entrepreneurs face tighter effective borrowing constraints. However, it also reduces inequality, since the tax on capital income is relatively more redistributive than a tax on wealth. They find the redistributive gain welfare-dominates the efficiency loss, and therefore prefer taxing capital income. Rotberg and Steinberg (2024) add endogenous tax evasion to a similar framework and find that taxes on wealth may be less attractive than taxes on capital income, because they generate more evasion as the taxed base is larger. Gerritsen et al. (2025) analyse optimal taxes on labour and capital income in a model where capital returns are higher for higher-ability individuals (type-dependent returns) or for wealthier individuals (scale-dependent returns). They characterise the Pareto-efficient structure of non-linear taxes on labour and capital income and show that taxes on capital income are positive. They do not compare taxes on capital income with taxes on wealth.[10] This literature shows that the taxes on wealth or capital income may differ once returns are heterogeneous and/or determined by entrepreneurship, but it does so in models without an explicit risk-return portfolio margin, as in the current paper.

Optimal taxation of capital income. A sixth strand is the large and mature literature on the desirability of taxes on capital income, to which this paper contributes. See Banks and Diamond (2010); Jacobs (2013); Bastani and Waldenström (2020); Scheuer and Slemrod (2021) for overviews. This literature mainly asks whether the (normal) return to saving should be taxed at all – with a zero tax as the classical benchmark (Atkinson and Stiglitz, 1976; Chamley, 1986; Judd, 1985b). Taxes on the normal return to capital may be optimal because (i) savings are relatively complementary to leisure (Corlett and Hague, 1953; Atkinson and Stiglitz, 1976; Erosa and Gervais, 2002; Jacobs and Boadway, 2014; Jacobs and Rusu, 2018), (ii) there is tax-base shifting between labour or entrepreneurial income and capital income (Christiansen and Tuomala, 2008; Reis, 2010), (iii) physical capital is a substitute of human capital (Jacobs and Bovenberg, 2010), (iv) saving preferences correlate positively with labour productivity (Diamond and Spinnewijn, 2011), (v) preferences for wealth itself may be heterogeneous (Saez and Stantcheva, 2018), (vi) risk-aversion correlates positively with labour productivity (Gerritsen and Dobbe, 2026), (vii) inheritances correlate positively with labour income (Cremer et al., 2001), (viii) there are dynamic inefficiencies in capital accumulation across overlapping generations (Ordover and Phelps, 1979; Atkinson and Sandmo, 1980), (ix) there are political constraints or a lack of commitment (Farhi et al., 2012; Scheuer and Wolitzky, 2016), (x) financial markets are imperfect due to borrowing constraints (Aiyagari, 1995; Hubbard and Judd, 1986), (xi) insurance markets are missing for idiosyncratic risk in labour income (Diamond and

[9] Guvenen et al. (2024) extend the comparison to book-value wealth taxation and innovation.

[10] In an extension, Gerritsen et al. (2025) derive optimal (ex post) taxes on wealth rather than capital income and they show that optimal wealth taxes are positive.

Mirrlees, 1978; Golosov et al., 2003; Conesa et al., 2009; Jacobs and Schindler, 2012). This paper analyses ex ante identical individuals having perfect access to capital markets (except for insurance markets for idiosyncratic return risk), with exogenous labour supply and human capital and no tax avoidance, political constraints or commitment issues. This is the reason why optimal taxes on the normal return to capital are zero. This paper contributes to the literature on optimal capital taxation by adding the portfolio margin and by showing that taxing the risk-premium is (weakly) desirable for efficiency reasons only, while a wealth subsidy offsets intertemporal distortions. Therefore, the results of this paper are complementary to the results of the optimal capital tax literature. The wealth subsidy should be lower, or should even turn into a tax, if taxing the normal return to capital would be optimal. Since the wealth tax is equivalent to a tax on the safe rate of return, this is equivalent to a differential income tax on safe and risky returns. As a methodological contribution to the capital tax literature, this paper derives dynamic Roy identities from the Bellman equation, which characterise the welfare effects of arbitrary changes in tax policy without solving the dynamic programme explicitly.

Tax neutrality and optimal portfolio choice. A seventh strand, in public finance and mathematical finance, studies the *neutrality* of capital taxes for valuation and portfolio choice. Bulow and Summers (1984) analyse the effective taxation of risky assets in an asset-pricing setting. They argue that taxing the risk premium has no social value, as in Gordon (1985) and Kaplow (1994), since the social cost of more risk in tax revenue exactly matches higher tax revenue. Sørensen (2005) formalises the rate-of-return allowance, under which shielding the normal return renders the shareholder-income tax neutral. Södersten (2020) identifies the capitalisation mechanism behind this result. Bjerksund and Schjelderup (2022) compare the valuation of an asset under a wealth tax and under a capital-income tax and establish conditions under which taxed and untaxed investors value it identically. Bond and Devereux (1995, 2003) show that cash-flow and allowance-for-corporate-equity taxes remain neutral under uncertainty, including with default risk. In continuous time, Ben Tahar et al. (2010) solve the Merton consumption–portfolio problem with a (realisation-based) capital-gains tax, and Frøseth (2026a,b,c) analyse proportional wealth taxes, and the contrast between flow (capital-income) and stock (wealth) taxes with a rate-of-return allowance, in a geometric-Brownian-motion/Fokker–Planck portfolio setting. These papers resemble the present paper most in terms of the modelling setup. This paper confirms the finding of this literature that taxes on capital income and on wealth are neutral for risk-taking and valuation. However, the papers in this literature ask a different question: *when* are tax instruments neutral or allocatively equivalent – taking behaviour as given or abstracting from the consumption–saving decision and from welfare? This paper solves for the optimising consumption–portfolio problem and asks: should taxes on capital income be preferred over taxes on wealth on welfare-economic grounds?

Policy discussions of wealth versus capital-income taxation. Finally, a policy-oriented literature weighs the two instruments directly. Abdel-Kader and De Mooij (2020), Boadway and Pestieau (2023), Bastani and Waldenström (2023) and Hebous et al. (2024) survey the choice between wealth and capital-income taxation. These papers come to the conclusion that taxes on capital income are to be preferred to taxes on wealth for both theoretical and practical

reasons. Scheuer and Slemrod (2021) survey the practical and conceptual differences between wealth taxes and capital-income taxes, emphasising that the two coincide only under uniform returns and diverge once returns are heterogeneous. They discuss the (de)merits of a wealth tax without taking a stance on its desirability.

This paper. Relative to the existing literature, this paper aims to compare the welfare-economic desirability of taxes on capital income and wealth in the canonical dynamic, stochastic Merton-Samuelson model of consumption and portfolio choice. In doing so, it confirms that many important insights from the one- and two-period models carry over to large investment horizons: the unit elastic response of risk-taking with respect to taxes, the social value of taxing the risk-premium under idiosyncratic risk and aggregate risk with state-contingent public goods, the neutrality of taxing the risk-premium with aggregate risk and state-contingent lump-sum taxes. Most importantly, in doing so, it demonstrates that taxes on capital income are weakly welfare superior to taxes on wealth.

3 Model

Taxes on capital income and wealth are introduced in the standard dynamic model of portfolio choice due to Merton (1969) and Samuelson (1969). In this model, infinitely lived individuals optimally decide on their intertemporal consumption-saving plan and their optimal portfolio choice between risky and safe assets.

3.1 States, expectations and aggregation

The model follows the discrete-time formulation of Samuelson (1969). The economy is populated by a continuum of ex ante identical, infinitely lived individuals, indexed by $i \in [0,1]$ and time $t = 0, 1, \cdots, \infty$. In each period, returns on risky assets depend on the state of the world, which is denoted by $\theta_t \in \Theta_t \equiv (-\infty, \infty)$. θ_t is not known ex ante. $\theta^t \equiv [\theta_0, \theta_1, \cdots, \theta_t]$ denotes the history of realised states θ up until t. The state θ_t is drawn from a conditional distribution with density $f(\theta_t \mid \theta^{t-1})$ given the history θ^{t-1}. The density of the state history θ^t is denoted by $f(\theta^t)$ and equals the product of the conditional densities: $f(\theta^t) = f(\theta^{t-1})f(\theta_t \mid \theta^{t-1}) = f(\theta_0)f(\theta_1 \mid \theta^0) \cdots f(\theta_t \mid \theta^{t-1})$. For notational simplicity, the dependence of all model variables – including all policy variables – on the state history θ^t is suppressed. In the same spirit, the conditioning history in the density is suppressed where no confusion arises, so that $f(\theta_t)$ is shorthand for $f(\theta_t \mid \theta^{t-1})$.

The derivations allow for intertemporal correlation of states in the case of aggregate return risk. However, in the case of idiosyncratic risk, it is assumed that states are independently distributed over time, so that idiosyncratic risk represents pure luck in asset returns, so that $f(\theta_t \mid \theta^{t-1}) = f(\theta_t)$ and $f(\theta^t) = f(\theta_0)f(\theta_1) \cdots f(\theta_t)$. Serially correlated idiosyncratic states would turn past luck into persistent, predictable return heterogeneity across individuals – similar to type-dependent returns – which would generate a redistributive case for taxing capital income from which this paper abstracts.[11]

[11] See Gerritsen et al. (2025) for the optimal tax on capital income or wealth with type-dependent returns.

The rate of return to the risky asset held by individual i in period t depends on the realisation of an individual-specific state θ_t^i. Every θ_t^i is drawn from the same conditional distribution with density $f(\theta_t \mid \theta^{t-1})$. Consequently, the expectations operator as of time t is identical across individuals: it conditions on the history θ^{t-1} and averages over the current-period state θ_t, which is realised only *after* the period-t consumption and portfolio decisions. Consequently, for any variable x that depends on θ_t, $\mathrm{E}_t[x] \equiv \int_{\Theta_t} x f(\theta_t \mid \theta^{t-1})\,\mathrm{d}\theta_t$. The following assumption specifies the conditions under which return risk is aggregate or idiosyncratic.

Assumption 1. *Risk in returns on assets is either idiosyncratic (I) or aggregate (A):*
***(I) Idiosyncratic risk**: the states θ_t^i are independently and identically distributed across individuals and over time, so that $f(\theta_t \mid \theta^{t-1}) = f(\theta_t)$, and the exact law of large numbers applies: $\int_0^1 g(\theta_t^i)\,\mathrm{d}i = \mathrm{E}_t[g(\theta_t)]$ almost surely for any integrable function $g(\cdot)$ (Judd, 1985a; Sun, 2006).*
***(A) Aggregate risk**: $\theta_t^i = \theta_t$ for all i, so that all individuals face the same common state realisation.*

Since each individual's decision problem depends only on the conditional distribution of its own return – which is $f(\theta_t \mid \theta^{t-1})$ and is the same under (I) and (A) – the entire description of (aggregate) saving and portfolio behaviour is identical in both risk cases. The individual index i is therefore suppressed wherever no confusion can arise. The two cases only differ in the government budget constraint, as will be shown below.

3.2 Individuals

Ex ante identical individuals maximise expected utility U, which is the expected discounted flow of sub-utilities from consumption c_t and public goods G_t in each year t:

$$U \equiv \mathrm{E}_0 \sum_{t=0}^{\infty} \frac{u(c_t) + \Gamma(G_t(\theta_t))}{(1+\beta)^t}, \quad u', \Gamma' > 0, \quad u'', \Gamma'' < 0, \quad u''' \geq 0, \quad \beta > 0, \tag{1}$$

where β is the individual's rate of time preference. The sub-utility functions $u(c_t)$ and $\Gamma(G_t(\theta_t))$ are strictly concave.[12] Utility is time-separable, as is completely standard in the literature. Therefore, optimal savings and portfolio choices can be derived using standard dynamic programming techniques. Since the horizon is infinite, the convergence condition of Samuelson (1969) is imposed to guarantee that the individual's problem remains well-defined.[13]

If there is idiosyncratic risk, the utility function $u(\cdot)$ is restricted to have constant relative risk aversion (CRRA) without much loss of generality, as in Samuelson (1969) and Merton (1969). By assuming CRRA utility under idiosyncratic risk, notational complexity is substantially reduced, since aggregation is simple as optimal consumption and saving are linear in wealth, and the optimal portfolio share is independent of wealth.[14]

[12] There is no bequest motive. The main insights would not change if a bequest motive were allowed for.

[13] In particular, expected discounted utility is finite for all feasible plans: either $u(\cdot)$ is bounded, or $u(\cdot)$ is CRRA with coefficient γ and the condition $\sup_t \frac{1}{1+\beta} \max_{\omega_t} \mathrm{E}_t[R_t^{1-\gamma}] < 1$ holds. Moreover, $\mathrm{E}_0 \sum_{t=0}^{\infty} (1+\beta)^{-t} \Gamma(G_t)$ is finite. As a result, the value function is the limit of its finite-horizon truncations, and the dynamic-programming approach below applies. See Alvarez and Stokey (1998) for a rigorous treatment of dynamic programming with unbounded, CRRA utility.

[14] All results of this paper would generalise to a general concave utility function, since the welfare analysis of tax reforms applies to each individual separately, irrespective of its wealth level. In the general case, portfolio

Public good provision $G_t(\theta_t)$ is state-dependent, as in Christiansen (1993), Schindler (2008) and Boadway and Spiritus (2025), so that public goods can be employed to (partly) insure against aggregate risk. Public goods are assumed to be separable in utility from private goods to avoid complicated second-best interactions between public-good provision, saving and risk-taking.

As regards the revelation of uncertainty, at the beginning of each period t the individual knows the history of past states θ^{t-1} and its assets a_t, and conditions its period-t saving and portfolio decisions on this information. The current state θ_t is realised only after these decisions have been made. Consequently, asset returns and – in the presence of aggregate risk – public goods provision or lump-sum taxes respond to θ_t, whereas period-t consumption is predetermined. The model is partial equilibrium and asset returns are exogenously given.

In each period, the individual earns exogenous labour income y_t. So there is no earnings inequality.[15] The individual can invest in risk-free government bonds b_t with a time-invariant fixed rate of return ρ or in risky stocks of firms e_t, which have a potentially time-varying stochastic rate of return $r_t(\theta_t)$, which depends on the state of the economy θ_t. In the worst possible state of the economy, the individual loses the entire value of the risky asset ($r(-\infty) \equiv -1$), while in the best possible state of the economy the risky asset has a potentially unbounded return ($r(\infty) \equiv \infty$). θ_t may reflect stock market risk, individual portfolio-investment ability, entrepreneurial talent or excess returns. If θ_t is stock market risk, it is most natural to assume that return risk is aggregate. If θ_t represents investment ability, entrepreneurial talent or pure excess returns, it is natural to assume that return risk is idiosyncratic.

τ_t^I is the linear tax rate on capital income and τ_t^A is the linear (ex ante) wealth tax. To demonstrate that the results of Gordon (1985), Kaplow (1994) and Hamilton (1987) can be obtained in the Samuelson-Merton framework, the government is allowed to levy state-contingent, non-individualised lump-sum taxes $T_t(\theta_t)$.

Total assets are denoted by $a_t \equiv b_t + e_t$. Initial assets are identical across individuals and exogenously given ($a_0 > 0$). A no-Ponzi-game condition is imposed that rules out explosive debts, i.e. the transversality condition $\lim_{s\to\infty}(1+\beta)^{-s}\mathrm{E}_t[u'(c_{t+s})a_{t+s}] = 0$ is assumed to hold, see the Appendix. The portfolio share of equities in total assets is denoted by $\omega_t \equiv e_t/a_t$, and the corresponding share of bonds is $1-\omega_t \equiv b_t/a_t$. Individuals cannot borrow at the safe rate to increase their equity holdings, i.e. take a leveraged equity position. Hence, bond holdings are non-negative ($b_t \geq 0$), so that $\omega_t \leq 1$.[16] The period t budget constraint of the individual in

shares and behavioural elasticities in the propositions would need to be replaced by saving-weighted averages of individual-specific portfolio shares and elasticities.

[15] Boadway and Spiritus (2025) have analysed the case with heterogeneity in labour earnings in a two-period framework with optimal portfolio choice and non-linear income taxes. They show that the optimal non-linear tax on labour income is the standard expression of Mirrlees (1971), since they assume homogeneous and weakly separable preferences. As a result, the optimality conditions for the optimal tax on labour income do not interact with the optimality conditions for the optimal taxes on capital income. Hence, the expressions for the taxes on capital income in this paper are comparable.

[16] This no-leverage constraint is assumed to be slack in the tax-reform analysis of Section 4, where portfolios are interior. It binds only in the analysis of optimal taxation under idiosyncratic risk in Sections 5 and 6. Therefore, the presentation of individual behaviour assumes it is slack as well. The constraint cannot be ignored with idiosyncratic risk. The Domar-Musgrave effect implies that a higher tax on capital income is met by a proportional increase in the equity position, $\omega_t = \omega_t^0(1-\tau_t^{I,0})/(1-\tau_t^I)$, which grows without bound as τ_t^I approaches one. The equity tax base $x_t = \omega_t s_t$, and thereby the revenue from taxing the risk premium, then also grows without bound, while the individual's expected utility remains unchanged. Social welfare would then

each period t for each state θ_t can thus be written as:[17]

$$c_t = y_t + a_t - s_t, \tag{2}$$

where $s_t \equiv y_t + a_t - c_t$ is end-of-period saving. The net return to savings R_t is given by

$$R_t \equiv (1 + (1 - \tau_t^I)\rho)(1 - \omega_t) + (1 + (1 - \tau_t^I)r_t(\theta_t))\omega_t - \tau_t^A. \tag{3}$$

Next-period assets are then given by

$$a_{t+1} \equiv R_t s_t - T_t(\theta_t). \tag{4}$$

The lump-sum tax does not appear in the period-t consumption budget, because it is contingent on the state θ_t, which is realised only after the consumption decision. Therefore, it is deducted from end-of-period resources, in the same way as capital returns accrue. Last period's state-contingent lump-sum tax thus reduces consumption possibilities at time t through $a_t \equiv R_{t-1}s_{t-1} - T_{t-1}(\theta_{t-1})$.

This modelling setup features no ex ante heterogeneity in wealth holdings, labour earnings or risk preferences.[18] In doing so, the model purely focuses on the efficiency properties of taxes on capital income and wealth.

The consumer's optimisation problem is solved using dynamic stochastic programming given that utility is time-separable. In particular, let $\mathcal{V}_t(a_t)$ be the value function at time t of total assets a_t:

$$\mathcal{V}_t(a_t) \equiv \max_{\{c_t,\omega_t\}} \mathrm{E}_t \sum_{s=t}^{\infty} \frac{u(c_s) + \Gamma(G_s(\theta_s))}{(1+\beta)^{s-t}}, \tag{5}$$

where $\max_{\{c_t,\omega_t\}}$ indicates that consumption c_t and portfolio weights ω_t are all chosen such that expected utility from $s = t$ onwards is maximised. For any period t, the following Bellman recursion is obtained – see Appendix A:

$$\mathcal{V}_t(a_t) \equiv \max_{c_t,\omega_t} u(c_t) + \mathrm{E}_t[\Gamma(G_t(\theta_t))] + \frac{1}{(1+\beta)}\mathrm{E}_t[\mathcal{V}_{t+1}((y_t + a_t - c_t)R_t - T_t(\theta_t))]. \tag{6}$$

Note that consumption at date t is deterministic, but public goods provision and the lump-sum tax may depend on the realisation of each state θ_t in the presence of aggregate risk. Consequently, one needs to take the expectation over public goods provision and the lump-sum tax as of time t. Since the lump-sum tax is non-individualised, each individual takes the state-contingent lump-sum tax $T_t(\theta_t)$ as given.

From the Bellman recursion follows that the period-t problem is basically a standard two-

have no maximum. At $\tau_t^I = 1$ the individual is moreover indifferent about the portfolio composition, so that the tax base would not even be determinate. From an economic point of view, a leveraged position requires a counterparty, and with ex ante identical individuals aggregate holdings of the safe asset must equal its net supply. Any finite bound $\omega_t \leq \bar{\omega}$ would serve equally well, in which case all results below hold with the pooled return $\rho + \bar{\omega}\pi_t$ instead of $\rho + \pi_t$.

[17] Like Samuelson (1969), some notational complexity is avoided by harmlessly assuming that consumption takes place at the end rather than at the beginning of each period.

[18] Cremer et al. (2001) have analysed the two-period case with heterogeneity in initial wealth in models without risk and their findings imply that the (normal) return to capital income should be taxed for redistributive reasons.

period portfolio problem in which the continuation value function $\mathcal{V}_{t+1}(\cdot)$ takes the place of second-period utility. In this precise sense, the infinite-horizon Merton-Samuelson model of this paper is an infinite collection of two-period portfolio-choice models, which is the precise reason why many of the insights from two-period models generalise to arbitrarily long horizons.

Maximisation of expected utility yields the following first-order conditions for optimal consumption and portfolio choices for each period t as long as the portfolio share of equities is interior:

$$u'(c_t) = \frac{1}{(1+\beta)} \mathrm{E}_t[\mathcal{V}'_{t+1}(\cdot) R_t], \tag{7}$$

$$\mathrm{E}_t[\mathcal{V}'_{t+1}(\cdot)(y_t + a_t - c_t)(1 - \tau_t^I)(r_t(\theta_t) - \rho)] = 0. \tag{8}$$

Equation (7) is the standard, stochastic Euler equation for consumption. Taxes on capital income and wealth may both distort saving choices. Equation (8) gives the standard optimality condition for the share of wealth that is invested in risky assets. Let the risk-premium on equities in year t be defined as $\pi_t \equiv -\frac{\mathrm{cov}_t[r_t(\theta_t), \mathcal{V}'_{t+1}(\cdot)]}{\mathrm{E}_t[\mathcal{V}'_{t+1}(\cdot)]}$, then equation (8) for optimal portfolio choice can be written as:

$$\mathrm{E}_t[r_t(\theta_t)] = \rho + \pi_t. \tag{9}$$

From this follows that optimal portfolio choice follows standard CAPM-logic: the expected return on risky assets is equal to the safe rate plus the risk premium. Portfolio choices are neither distorted by the tax on capital income nor by the tax on wealth. Hence, both tax instruments are neutral with respect to risk-taking, so that the optimal degree of risk taking is not distorted.[19] Intuitively, the opportunity cost of investing in equities is giving up a safe return on bonds to obtain a (on average) higher but more risky return on equities. Since costs and benefits of investing more resources in equities are symmetrically affected by both taxes on capital income and wealth, these taxes do not affect optimal risk-taking. This neutrality result is the dynamic counterpart of well-known results in the literature. Domar and Musgrave (1944), Mossin (1968), Stiglitz (1969) and Sandmo (1969) show in one- and two-period models that a proportional tax on all returns with full loss offset leaves the optimal risk exposure unaffected: individuals scale up their holdings of risky assets so as to restore their pre-tax exposure to risk.

The neutrality literature in public finance and mathematical finance establishes the same result for valuation. Sørensen (2005), Södersten (2020) and Bond and Devereux (1995, 2003) show that a tax on capital income with a rate-of-return allowance – which taxes only the risk premium – is neutral for investment and portfolio decisions, because the tax on the risk premium is capitalised in asset prices at the same rate as the risk premium before tax. Bjerksund and Schjelderup (2022) and Frøseth (2026a,b) derive conditions under which taxed and untaxed investors value assets identically under wealth and capital-income taxes. Eq. (8) shows that these findings are obtained in the infinite-horizon Merton-Samuelson model: both the tax on capital income and the tax on wealth multiply the excess return $r_t(\theta_t) - \rho$ by a factor that does not depend on the state θ_t, so that neither tax drives a wedge between the marginal cost and the marginal benefit of bearing risk.[20]

[19] From Stiglitz (1969), Atkinson and Stiglitz (1980) and Sandmo (1985) follows that the optimal degree of risk taking is dependent on whether there are taxes on capital income or wealth. However, even though risk-taking is affected, it is not *distorted* because there is no wedge between social and private returns to risk-taking.

[20] Note, however, that neutrality with respect to the *optimal* degree of risk-taking does not imply neutrality

The first-order conditions for saving and portfolio choice give the optimal policy rules for consumption c_t^* and ω_t^* at each time t that depend on the entire history of shocks θ^{t-1} up until $t-1$:

$$c_t^* \equiv c(a_t, \theta^{t-1}), \quad \omega_t^* \equiv \omega(a_t, \theta^{t-1}). \tag{10}$$

If the shocks θ_t are independently distributed over time, the policy rules are (time-dependent) functions only of the stock of wealth a_t at time t:

$$c_t^* \equiv c_t(a_t), \quad \omega_t^* \equiv \omega_t(a_t). \tag{11}$$

3.3 Government

The government collects taxes on capital income and wealth, it issues government debt d_t and it provides public goods $G_t(\theta_t)$, all at time t. Initial debt is given by d_0. The government satisfies the no-Ponzi condition $\lim_{s\to\infty}(1+\rho)^{-s}d_{t+s} = 0$, so that the intertemporal government budget constraint always holds. The timing of the government budget follows the same conventions as the timing of the individual budget constraint. Individuals are taxed on the returns to period-t saving when these returns are realised, i.e. at the beginning of period $t+1$. Next-period asset holdings are savings multiplied with net asset returns minus lump-sum taxes, i.e. $a_{t+1} = R_t s_t - T_t(\theta_t)$. Symmetrically, the government collects the revenue from the period-t tax bases at the beginning of period $t+1$, so that this revenue enters the government budget constraint of period $t+1$. The government budget constraint for every $t \geq 0$ is therefore given by:

$$d_{t+2} - d_{t+1} = \rho d_{t+1} + G_{t+1}(\theta_{t+1}) - \tau_t^I(\rho + (r_t(\theta_t) - \rho)\omega_t)s_t - \tau_t^A s_t - T_t(\theta_t). \tag{12}$$

Period-0 public goods are financed by initial debt issuance: $d_1 = (1+\rho)d_0 + G_0$. Under aggregate risk it is not necessary to introduce state-contingent government debt if there are state-contingent lump-sum taxes. The reason is that allowing for state-contingent debt joint with a deterministic lump-sum tax is economically equivalent to assuming state-contingent lump-sum taxes.

The tax base for taxes on capital income or wealth is end-of-period savings $s_t \equiv y_t + a_t - c_t$. The government can smooth aggregate risks intratemporally – by adjusting the level of public goods provision – and intertemporally – by adjusting government debt. From the government budget constraint follows that the base of the capital income tax is the safe rate of return on all assets ρs_t plus the risk premium on equities $(r_t(\theta_t) - \rho)\omega_t s_t$, whereas the base of the (ex ante) wealth tax is given by end-of-period saving s_t. Individualised lump-sum taxes and taxes on labour earnings are ruled out. The latter are also lump-sum taxes, since labour income is exogenous. State-contingent adjustments of $T_t(\theta_t)$ are considered only in the neutrality analysis of Proposition 3. In all other policy experiments and in the analysis of optimal taxation, the state-contingent lump-sum tax is held constant.

The three cases analysed below differ in the instrument that closes the government budget in each state, and therefore in the set of instruments that the government optimises

with respect to the *level* of risk-taking: the Domar-Musgrave effect implies that the portfolio share responds to taxes with unit elasticity, see below.

over. Under idiosyncratic risk the government optimises $\{\tau_t^I, \phi_t, G_{t+1}, d_{t+2}\}$ and the lump-sum tax T_t is exogenous; under aggregate risk with state-contingent lump-sum taxes it optimises $\{\tau_t^I, \phi_t, T_t(\theta_t), G_{t+1}(\theta_{t+1}), d_{t+2}\}$; and under aggregate risk with state-contingent public goods it optimises $\{\tau_t^I, \phi_t, G_{t+1}(\theta_{t+1}), d_{t+2}\}$ with $T_t(\theta_t)$ held fixed. These differences are deliberate. A lump-sum instrument that is unrestricted in every state is non-distortionary by construction, so a government that commands one alongside the capital taxes attains the first best regardless of how it taxes capital, and the comparison between the two capital tax bases becomes vacuous. Exactly one rebate channel is therefore left open in each case, and the case distinction is which channel that is: lower deterministic taxes under idiosyncratic risk, state-contingent lump-sum taxes in the experiment of Gordon (1985) and Kaplow (1994), and state-contingent public goods in the experiment of Christiansen (1993), Schindler (2008) and Boadway and Spiritus (2025).

The cases are consequently not nested, and the case with state-contingent lump-sum taxes is not the case with idiosyncratic risk augmented by a free lump-sum instrument. Under idiosyncratic risk the aggregate tax base is deterministic by the exact law of large numbers, so that there is no aggregate state on which a non-individualised lump-sum tax could be conditioned: $T_t(\theta_t) = T_t$. What Proposition 6 adds relative to Proposition 5 is therefore not the lump-sum instrument as such, but its *state-contingency*, which is meaningless when there is no aggregate risk to condition on. The differences in the results across the three propositions track these differences in the instrument sets exactly. With one lump-sum instrument per state the government has as many instruments as margins, attains the first best, and the tax on capital income becomes indeterminate. With one instrument fewer it does not, and the tax on capital income is pinned down – at the corner $\tau_t^I = 1$ under idiosyncratic risk, and in the interior when aggregate risk is returned through public goods.

Under aggregate risk – case (A) of Assumption 1 – all individuals remain identical ex post, so that individual and per-capita variables coincide, and the government budget constraint in eq. (12) is stochastic: tax revenue depends on the realisation of the common shock θ_t. If all risk is idiosyncratic – case (I) of Assumption 1 – the government pools the independent return risks of individuals. The tax bases in eq. (12) then represent the cross-sectional aggregates: revenue from the capital-income tax equals $\tau_t^I \int_0^1 (\rho + (r_t(\theta_t^i) - \rho)\omega_t) s_t^i \,\mathrm{d}i$, where s_t^i denotes saving of individual i. Two properties make the aggregate tax base deterministic. First, portfolio shares and saving are chosen *before* the current-period states θ_t^i are realised, and the portfolio share ω_t is identical across individuals under CRRA-utility. Second, the exact law of large numbers of Assumption 1 then implies $\int_0^1 (r_t(\theta_t^i) - \rho)\omega_t s_t^i \,\mathrm{d}i = \pi_t \omega_t s_t$ almost surely, where $s_t \equiv \int_0^1 s_t^i \,\mathrm{d}i$ henceforth denotes average saving, and the expected excess return equals the risk premium ($\pi_t = \mathrm{E}_t[r_t(\theta_t)] - \rho$) by the portfolio optimality condition. Average saving is deterministic, since the entire cross-sectional wealth distribution is deterministic under the exact law of large numbers. Moreover, under idiosyncratic risk there is no aggregate state on which the non-individualised lump-sum tax can be conditioned, so that the lump-sum tax is deterministic as well ($T_t(\theta_t) = T_t$). Hence, tax revenue becomes deterministic for all t:

$$d_{t+2} - d_{t+1} = \rho d_{t+1} + G_{t+1} - \tau_t^I(\rho + \pi_t\omega_t)s_t - \tau_t^A s_t - T_t. \tag{13}$$

From this follows that the tax base of the capital income tax is the normal return on all assets

ρs_t plus the risk premium on equity $\pi_t \omega_t s_t$, and the tax base of the ex ante wealth tax is end-of-period saving s_t.

4 Tax reforms

To explore the desirability of capital income taxes over wealth taxes, a specific tax reform will be analysed, which delivers sharp comparative statics. In particular, an exogenously given tax system is considered with positive taxes on capital income and wealth, which does not need to be optimal. The following policy reform is analysed: the capital income tax is raised in year t with $\mathrm{d}\tau_t^I > 0$, and the wealth tax is reduced in that same year with $\mathrm{d}\tau_t^A < 0$, such that the intertemporal price of consumption, i.e. the net, after-tax safe rate of return does not change. In this tax reform, the government shifts the burden of taxes on capital from the safe return towards the risk premium. The following lemma provides the specifics of the tax reform studied in this paper.

Lemma 1 (Tax reform)**.** *The net, after-tax safe rate of return is given by* $(1-\tau_t^I)\rho - \tau_t^A$*. Any reform of taxes on wealth and capital income that leaves the intertemporal price of consumption unaffected satisfies* $\mathrm{d}\tau_t^A = -\rho \mathrm{d}\tau_t^I$*.*

The main question is whether a tax reform that raises the tax on capital income and lowers the tax on wealth raises utility or government revenue. If it does, then a tax on capital income is welfare superior to a tax on wealth.

4.1 Dynamic stochastic Roy's identity

To sign the welfare effects of the tax reforms, and to derive optimal tax policies, the analysis exploits a dynamic stochastic version of Roy's identity with respect to all policy instruments. It is derived from the observation that the value function $\mathcal{V}_t(a_t)$ can be directly related to the indirect utility function. In particular, the indirect utility function is given by the value function at time $t = 0$:

$$U = \mathcal{V}_0(a_0) \equiv \max_{\{c_t, \omega_t\}} \mathrm{E}_0 \sum_{s=0}^{\infty} \frac{u(c_s) + \Gamma(G_s(\theta_s))}{(1+\beta)^s}, \tag{14}$$

where consumption c_t and portfolio weights ω_t are all chosen such that expected utility from $s = 0$ onwards is maximised. From this expression, the derivatives of the value function can be found with respect to all relevant tax policy variables, as shown in the next Lemma. The derivations in the Appendix also make extensive use of the net safe after-tax return factor $\phi_t \equiv 1 + (1-\tau_t^I)\rho - \tau_t^A$, which remains constant under the tax reforms of Lemma 1; the derivatives with respect to ϕ_t are therefore included as well.

Lemma 2 (Dynamic-stochastic Roy's identity)**.** *(i) Assuming the portfolio choice is interior,*

the period-t recursion of the Bellman equation has the following partial derivatives for all t:[21]

$$\frac{\partial \mathcal{V}_t(a_t)}{\partial a_t} = \frac{1}{(1+\beta)} \mathrm{E}_t[\mathcal{V}'_{t+1}(\cdot) R_t] = u'(c_t), \tag{15}$$

$$\frac{\partial \mathcal{V}_t(a_t)}{\partial \tau_t^I} = -\frac{\mathrm{E}_t[\mathcal{V}'_{t+1}(\cdot)]}{(1+\beta)} (y_t + a_t - c_t)\rho, \tag{16}$$

$$\frac{\partial \mathcal{V}_t(a_t)}{\partial \tau_t^A} = -\frac{\mathrm{E}_t[\mathcal{V}'_{t+1}(\cdot)]}{(1+\beta)} (y_t + a_t - c_t), \tag{17}$$

$$\frac{\partial \mathcal{V}_t(a_t)}{\partial \phi_t} = \frac{s_t \, \mathrm{E}_t[\mathcal{V}'_{t+1}(\cdot)]}{1+\beta} = \frac{u'(c_t)}{\phi_t} s_t, \tag{18}$$

$$\left.\frac{\partial \mathcal{V}_t(a_t)}{\partial \tau_t^I}\right|_{\phi_t} = 0, \tag{19}$$

$$\frac{\partial \mathcal{V}_t(a_t)}{\partial G_t(\theta_t)} = \Gamma'(G_t(\theta_t)) f(\theta_t), \tag{20}$$

$$\frac{\partial \mathcal{V}_t(a_t)}{\partial T_t(\theta_t)} = -\frac{1}{(1+\beta)} \mathcal{V}'_{t+1}(\cdot) f(\theta_t) = -\frac{u'(c_{t+1}(\theta_t)) f(\theta_t)}{1+\beta}, \tag{21}$$

where the derivative in eq. (19) is taken at a fixed ϕ_t, i.e. at the tax reform $\mathrm{d}\tau_t^A = -\rho \mathrm{d}\tau_t^I$.
(ii) For any policy instrument z_t dated t and set in history θ^{t-1}, the derivative of lifetime utility is given by

$$\frac{\partial \mathcal{V}_0(a_0)}{\partial z_t} = \frac{f(\theta^{t-1})}{(1+\beta)^t} \frac{\partial \mathcal{V}_t(a_t)}{\partial z_t}, \tag{22}$$

where $f(\theta^{t-1})$ is the density of the history θ^{t-1}. Under idiosyncratic risk, where policy is deterministic, the density factor is absent. Consequently, using $f(\theta^{t-1}) f(\theta_t) = f(\theta^t)$, the derivatives of lifetime expected utility with respect to the tax instruments are given by:

$$\frac{\partial \mathcal{V}_0(a_0)}{\partial \phi_t} = \frac{f(\theta^{t-1})}{(1+\beta)^t} \frac{u'(c_t)}{\phi_t} s_t, \tag{23}$$

$$\left.\frac{\partial \mathcal{V}_0(a_0)}{\partial \tau_t^I}\right|_{\phi_t} = 0, \tag{24}$$

$$\frac{\partial \mathcal{V}_0(a_0)}{\partial G_t(\theta_t)} = \frac{f(\theta^t)}{(1+\beta)^t} \Gamma'(G_t(\theta_t)), \tag{25}$$

$$\frac{\partial \mathcal{V}_0(a_0)}{\partial T_t(\theta_t)} = -\frac{f(\theta^t)}{(1+\beta)^{t+1}} u'(c_{t+1}(\theta_t)). \tag{26}$$

Proof See Appendix B.

4.2 Comparative statics

To sign the welfare effects of the tax reform in Lemma (1), the comparative statics of saving and portfolio choice with respect to this tax reform are derived. These comparative statics are given in the next Lemma.

[21] If the no-leverage constraint binds ($\omega_t = 1$), the first-order condition for portfolio choice holds with inequality, $\Delta_t \equiv \mathrm{E}_t[\mathcal{V}'_{t+1}(\cdot)(r_t(\theta_t) - \rho)] \geq 0$, and eqs. (16), (18) and (19) generalise to $\partial \mathcal{V}_t(a_t)/\partial \tau_t^I = -s_t \left(\rho \mathrm{E}_t[\mathcal{V}'_{t+1}(\cdot)] + \Delta_t\right)/(1+\beta)$, $\partial \mathcal{V}_t(a_t)/\partial \phi_t = s_t \mathrm{E}_t[\mathcal{V}'_{t+1}(\cdot)]/(1+\beta)$ (the second equality in eq. (18) then requires $(1-\tau_t^I)\Delta_t = 0$, which holds at $\tau_t^I = 1$), and $\partial \mathcal{V}_t(a_t)/\partial \tau_t^I\big|_{\phi_t} = -\omega_t s_t \Delta_t/(1+\beta) \leq 0$. Eqs. (15), (17), (20) and (21) are unaffected. The corresponding expressions for lifetime utility follow from eq. (22). These expressions are used in the proof of Proposition 5, where the no-leverage constraint binds at the optimum.

Lemma 3 (Comparative statics tax reform)**.** *Assume that portfolio choice is interior ($\omega_t < 1$), that $\tau_t^I < 1$ and that $s_t > 0$. Consider an increase in the tax on capital income and a reduction in the ex ante wealth tax in any period t such that expected utility remains fixed: $\mathrm{d}\tau_t^A = -\rho \mathrm{d}\tau_t^I$. This tax reform has the following comparative statics:*

1. *The intertemporal price of consumption in period t does not change ($\mathrm{d}((1 + (1 - \tau_t^I)\rho) - \tau_t^A) = 0$),*
2. *Total saving and consumption choices in period t are not affected ($\mathrm{d}s_t = \mathrm{d}c_t = 0$),*
3. *The portfolio share of risky assets increases such that the same risk-return trade-off is obtained in period t,*
4. *The policy elasticities of saving and portfolio weights under the tax reform are given by:*

$$\varepsilon_{s\tau^I} \equiv -\frac{\mathrm{d}s_t}{\mathrm{d}\tau_t^I}\frac{(1-\tau_t^I)}{s_t} = 0, \quad \varepsilon_{\omega\tau^I} \equiv \frac{\mathrm{d}\omega_t}{\mathrm{d}\tau_t^I}\frac{(1-\tau_t^I)}{\omega_t} = 1. \tag{27}$$

Proof See Appendix C.1.

The policy elasticities capture the behavioural response of saving and portfolio choice to the tax reform $\mathrm{d}\tau_t^A = -\rho \mathrm{d}\tau_t^I$. These elasticities are valid only under this particular tax reform. The intuition for the comparative statics in portfolio choice follows from Domar and Musgrave (1944), Mossin (1968), Stiglitz (1969), Sandmo (1969). Under the particular tax reform, the tax burden is shifted from the safe rate of return to the risk premium. As a result, individuals reshuffle their portfolios by increasing the share of equities in response to the higher tax on capital income to obtain the same net-return-risk combination. That is why the elasticity of the portfolio share with respect to the net-of-tax rate on capital income (i.e. $1 - \tau_t^I$) is one (in absolute value). This behavioural response is referred to as the *Domar and Musgrave (1944)-effect*.

Since the intertemporal price of consumption is not affected by the tax reform, the individual does not want to change intertemporal consumption behaviour. Hence, saving and consumption choices are not affected. Indeed, if the consumer can achieve the same probability distribution of future consumption as before by a simple rearrangement of his portfolio, there is no reason to change the level of saving and, thereby, consumption choices (Sandmo, 1969; Schindler, 2008). As a result, the comparative statics of the particular tax reform are very sharp and allow us to sign the welfare effects of the reform.[22]

The policy elasticities are independent of whether risk in capital income is idiosyncratic or aggregate – except for the case that risk is aggregate and there are state-contingent lump-sum taxes, which will be discussed later. The reason is, first and most importantly, that individuals take return realisations as given, irrespective of whether risk is idiosyncratic or aggregate from an aggregate perspective. Second, private and public consumption are separable in utility, so that state-contingent public goods do not interact with savings and portfolio choices.[23]

[22] The comparative statics of this particular tax reform are identical to the comparative statics of a tax on the risk-premium alone, as demonstrated by Boadway and Spiritus (2025). Intuitively, the joint tax reform of raising the capital income tax and lowering the wealth tax is equivalent to raising the tax on the risk-premium.

[23] The latter is also (trivially) true for state-contingent lump-sum taxes.

On a technical note, the appendix shows that comparative statics are global, and no restrictions on second-order conditions are imposed beyond assuming strict concavity. Therefore, all comparative statics also apply to non-marginal reforms as long as they keep the net intertemporal price of consumption constant. The reason is that taxes *only* affect behaviour through the after-tax safe return. As long as it remains constant, which is the case in the particular policy reforms analysed, the comparative statics are very simple.[24]

4.3 Welfare effects tax reform – idiosyncratic risk

The effects of the tax reform in Lemma (1) will be analysed under two different assumptions regarding risk in capital income: all risk is idiosyncratic or all risk is aggregate. This section starts with the case in which all risk is idiosyncratic and government revenue is deterministic.

Denote net government revenue (net debt reduction) in period $t+1$ – when the taxes on period-t bases are collected – as $\mathcal{R}_{t+1} \equiv (1+\rho)d_{t+1} - d_{t+2}$. The deterministic period-$t+1$ government budget constraint in eq. (13) can then be rewritten as:

$$\mathcal{R}_{t+1} \equiv (1+\rho)d_{t+1} - d_{t+2} = \tau_t^I(\rho + \pi_t\omega_t)s_t + \tau_t^A s_t + T_t - G_{t+1}. \tag{28}$$

It will be shown that the tax reform of Lemma (1) keeps expected utility fixed. As a result, the total welfare effect of the tax reform can be signed by calculating the change in public resources $\mathrm{d}\mathcal{R}_{t+1}/\mathrm{d}\tau_t^I$. This welfare effect is identical to rebating the increase in public revenue as a deterministic lump-sum transfer to the individual (i.e. a reduction in the lump-sum tax T_t). This yields the first main proposition of the paper.

Proposition 1 (Superiority of taxes on capital income over taxes on wealth)**.** *Starting from an initial tax system with positive taxes on capital income and on wealth, presuming that all risk in capital income is idiosyncratic, if the tax on capital income is raised, and the tax on wealth is lowered such that* $\mathrm{d}\tau_t^A = -\rho\mathrm{d}\tau_t^I$*, then social welfare in period t increases according to:*

$$\frac{\mathrm{d}\mathcal{R}_{t+1}/\mathrm{d}\tau_t^I}{s_t} = \pi_t\omega_t - \frac{(\tau_t^I(\rho+\pi_t\omega_t)+\tau_t^A)}{1-\tau_t^I}\varepsilon_{s\tau^I} + \frac{\tau_t^I}{1-\tau_t^I}\pi_t\omega_t\varepsilon_{\omega\tau^I} = \frac{\pi_t\omega_t}{1-\tau_t^I} > 0, \tag{29}$$

where the elasticities follow from Lemma (3).

Proof See Appendix D.

From Proposition (1) follows that taxes on capital income are to be preferred over taxes on wealth. The intuition is that a tax reform that raises the tax on capital income and lowers the tax on wealth implies a shift in the tax burden away from the safe rate of return towards the risk premium. This effect is captured by the first term in the right-hand side ($\pi_t\omega_t$). By taxing the risk premium, the efficiency of the tax system is increased. The underlying reason is that there is a missing market for insuring idiosyncratic return risk. The government provides social

[24] Lemma (3) is the infinite-horizon counterpart of Lemma 1 in Boadway and Spiritus (2025), who show in a two-period model that a tax on excess returns leaves labour supply and saving unchanged and yields a unit elasticity of risky assets with the net-of-tax rate. Their complex proof relies on totally differentiating the individual's first-order conditions and applying Cramer's rule. This is not required in this paper.

insurance by pooling these risks and returning the revenue in (lower) deterministic taxes. Doing so alleviates this market failure, while not generating additional distortions, neither in saving nor in risk-taking. The welfare effect is larger if the risk premium is bigger and the portfolio share is larger.[25]

Distortions in saving and portfolio choices are captured by the second and third terms on the right-hand side. There is no effect on saving ($\varepsilon_{s\tau^I} = 0$), since the intertemporal price of consumption does not change. Moreover, while risk-taking is not distorted – taxes on wealth and capital income affect safe and risky assets symmetrically –, it is enhanced via the Domar-Musgrave effect ($\varepsilon_{\omega\tau^I} = 1$). Indeed, as the government provides better insurance against risk in capital incomes, individuals respond by increasing their portfolio share of risky assets. As a result of larger risk-taking, the government also collects more revenue from taxing the risk-premium.

From this analysis follows the important insight that the one- and two-period portfolio choice models can be fully generalised to the infinite-horizon setting. Indeed, the Merton-Samuelson model reveals that – due to its recursive structure – the infinite-horizon model is like a large collection of two-period models.

An argument sometimes used in the debate on taxes on wealth versus capital income is that wealth is amortised capital income earned in previous periods of the life-cycle. Hence, as the time-horizon of the investor becomes longer, the distinction between taxes on wealth and on capital income gradually vanishes over time as wealth in later periods is more and more cumulated capital income. The wealth tax would then start to resemble more and more a tax on capital income. However, the analysis above reveals that this intuition is misleading. Given the recursive structure, the choice between taxing wealth or capital income is solely determined by the desire to tax the risk premium. The relative benefits of doing so increase linearly in the stock of wealth. Hence, as the stock of wealth increases, so does the benefit of taxing the risk premium. The merits of taxing wealth are thus *not* driven by the benefits of taxing previously earned capital incomes.

4.4 Ex post wealth taxes

So far, only ex ante taxes on wealth τ_t^A have been analysed. But what can be said about ex post taxes on wealth τ_t^P? One may argue that ex post wealth taxes include the risk-premium in the tax base. Hence, ex post taxes may still be more desirable than ex ante taxes on wealth, which do not include the risk-premium in the tax base at all. Therefore, this section explores the role of ex post wealth taxes along the same lines as ex ante wealth taxes in the model with idiosyncratic risk.

With ex post wealth taxes τ_t^P, the net return factor R_t is modified for each realisation θ_t:

$$R_t \equiv (1-\tau_t^P)[(1+(1-\tau_t^I)\rho)(1-\omega_t)+(1+(1-\tau_t^I)r_t(\theta_t))\omega_t]. \tag{30}$$

Like before, tax reforms will be analysed that raise the capital income tax and lower the wealth

[25] This finding is related to Eaton and Rosen (1980) and Varian (1980) (and others) in the context of taxes on labour earnings. They have shown that taxes on labour income are efficient to pool earnings risk and distortions in incentives are optimally balanced against benefits of insurance.

tax such that the net, after-tax safe rate of return does not change. The next Lemma provides this tax reform.

Lemma 4 (Tax reform ex post wealth tax)**.** *The net, after-tax safe rate of return is given by* $(1-\tau_t^P)(1+(1-\tau_t^I)\rho)-1$*. Any reform of taxes on wealth and capital income that leaves the intertemporal price of consumption unaffected satisfies* $\mathrm{d}\tau_t^P = -\frac{(1-\tau_t^P)\rho}{1+(1-\tau_t^I)\rho}\mathrm{d}\tau_t^I$*.*

The comparative statics of the tax reform on saving and portfolio behaviour are derived in the next Lemma.

Lemma 5 (Comparative statics tax reform ex post wealth tax)**.** *Assume that portfolio choice is interior (*$\omega_t < 1$*), that* $\tau_t^I < 1$*,* $\tau_t^P < 1$ *and that* $s_t > 0$*. Consider an increase in the tax on capital income and a reduction in the ex post wealth tax in any period* t *such that expected utility remains fixed:* $\mathrm{d}\tau_t^P = -\frac{(1-\tau_t^P)\rho}{1+(1-\tau_t^I)\rho}\mathrm{d}\tau_t^I$*. This tax reform has the following comparative statics:*

1. *The intertemporal price of consumption in period* t *does not change (*$\mathrm{d}((1-\tau_t^P)(1+(1-\tau_t^I)\rho)) = 0$*),*
2. *Total saving and consumption choices in period* t *are not affected (*$\mathrm{d}s_t = \mathrm{d}c_t = 0$*),*
3. *The portfolio share of risky assets increases such that the same risk-return trade-off is obtained in period* t*,*
4. *The policy elasticities of saving and portfolio weights under the tax reform are given by:*

$$\varepsilon_{s\tau^I} \equiv -\frac{\mathrm{d}s_t}{\mathrm{d}\tau_t^I}\frac{(1-\tau_t^I)}{s_t} = 0, \quad \varepsilon_{\omega\tau^I} \equiv \frac{\mathrm{d}\omega_t}{\mathrm{d}\tau_t^I}\frac{(1-\tau_t^I)}{\omega_t} = \frac{1}{1+(1-\tau_t^I)\rho}. \tag{31}$$

Proof See Appendix C.1.

The government budget constraint under ex post wealth taxes, with idiosyncratic risk, can be written as:

$$d_{t+2} - d_{t+1} = \rho d_{t+1} + G_{t+1} - \tau_t^I(\rho + \pi_t\omega_t)s_t - \tau_t^P(1+(1-\tau_t^I)(\rho+\pi_t\omega_t))s_t - T_t. \tag{32}$$

Armed with the definition of the tax reform and the comparative statics, the first Proposition can be generalised to ex post wealth taxes in the next Proposition.

Proposition 2 (Superiority of taxes on capital income over ex post taxes on wealth)**.** *Starting from an initial tax system with positive taxes on capital income and ex post taxes on wealth, presuming that all risk in capital income is idiosyncratic, if the tax on capital income is raised, and the ex post tax on wealth is lowered such that* $\mathrm{d}\tau_t^P = -\frac{(1-\tau_t^P)\rho}{1+(1-\tau_t^I)\rho}\mathrm{d}\tau_t^I$*, then social welfare in*

period t increases according to:

$$\begin{aligned}\frac{\mathrm{d}\mathcal{R}_{t+1}/\mathrm{d}\tau_t^I}{s_t} &= \frac{(1-\tau_t^P)\pi_t\omega_t}{1+(1-\tau_t^I)\rho} \\ &\quad -\frac{(\tau_t^I(\rho+\pi_t\omega_t)+\tau_t^P(1+(1-\tau_t^I)(\rho+\pi_t\omega_t)))}{(1-\tau_t^I)}\varepsilon_{s\tau^I} \\ &\quad +\frac{(\tau_t^I+\tau_t^P(1-\tau_t^I))}{(1-\tau_t^I)}\pi_t\omega_t\varepsilon_{\omega\tau^I} \\ &= \frac{\pi_t\omega_t}{(1-\tau_t^I)(1+(1-\tau_t^I)\rho)} > 0,\end{aligned} \tag{33}$$

where the elasticities follow from Lemma (5).

Proof See Appendix E.

The welfare effect of the tax reform with ex post wealth taxes turns out to be remarkably simple: it equals the welfare effect with ex ante wealth taxes in Proposition 1 ($\pi_t\omega_t/(1-\tau_t^I)$) divided by the after-tax gross safe return ($1+(1-\tau_t^I)\rho$). The reason is that the ex post wealth tax is levied on end-of-period wealth inclusive of accrued returns, so that one unit of ex post wealth tax corresponds to $1+(1-\tau_t^I)\rho$ units of ex ante wealth tax. The interpretation is otherwise exactly the same as in Proposition 1: replacing the ex post wealth tax with a tax on capital income is unambiguously welfare-improving, since it moves the tax burden towards the risk premium, and it enhances risk-taking – albeit less than one-for-one, since a reduction in the ex post wealth tax itself restores part of the individual's net risk exposure. Indeed, the portfolio elasticity is smaller than one ($\varepsilon_{\omega\tau^I} = [1+(1-\tau_t^I)\rho]^{-1} < 1$), since the individual reshuffles its portfolio less than one-for-one with the capital income tax. Consequently, the basic logic why a tax on capital income is welfare superior to a tax on wealth extends to the case with ex post wealth taxes.

4.5 Welfare effects tax reform – aggregate risk and state-contingent lump-sum taxes

In this and the next subsection, the case with aggregate risk is explored. As a result, government revenue is stochastic. The change in expected tax revenue can then no longer serve as a statistic for the welfare effects of the tax reform: the government budget constraint needs to hold for each state realisation θ_t, so that some policy instrument has to absorb the change in revenue in each state of the world. In addition, the wealth tax is again ex ante.

This subsection considers the classic policy experiment of Gordon (1985) and Kaplow (1994): the government keeps public goods provision constant, and balances its budget in each state θ_t by adjusting state-contingent, non-individualised lump-sum taxes (or transfers) $T_t(\theta_t)$. The next subsection considers the case in which the government balances its budget by adjusting state-contingent public goods provision instead.

In the case with aggregate risk and state-contingent lump-sum taxes, the comparative statics of the tax reform are different from those with idiosyncratic risks or aggregate risk with state-contingent public goods. The next Lemma derives these comparative statics.

Lemma 6 (Comparative statics tax reform with aggregate risk and state-contingent taxes)**.** *Consider an increase in the tax on capital income and a reduction in the tax on wealth in any period t such that expected utility remains fixed: $\mathrm{d}\tau_t^A = -\rho \mathrm{d}\tau_t^I$, and the government rebates any stochastic change in revenue by adjusting the state-contingent lump-sum tax such that $\mathrm{d}\mathcal{R}_{t+1} = 0$, and $\mathrm{d}T_t(\theta_t) \gtrless 0$ for any θ_t. This tax reform has the following comparative statics:*[26]

1. *The intertemporal price of consumption in period t does not change ($\mathrm{d}((1 + (1 - \tau_t^I)\rho) - \tau_t^A) = 0$),*
2. *Total saving and consumption choices in period t are not affected ($\mathrm{d}s_t = \mathrm{d}c_t = 0$),*
3. *Risk-taking is not affected,*
4. *The policy elasticities of saving and portfolio weights under the tax reform are given by:*

$$\varepsilon_{s\tau^I} \equiv -\frac{\mathrm{d}s_t}{\mathrm{d}\tau_t^I}\frac{(1-\tau_t^I)}{s_t} = 0, \quad \varepsilon_{\omega\tau^I} \equiv \frac{\mathrm{d}\omega_t}{\mathrm{d}\tau_t^I}\frac{(1-\tau_t^I)}{\omega_t} = 0. \tag{34}$$

Proof See Appendix C.2.

Under the tax reform analysed, the individual thus makes the same consumption and portfolio choices as before the reform. In contrast to cases with idiosyncratic risk or aggregate risks and state-contingent public goods, the Domar-Musgrave effect now vanishes. The individual does not raise its portfolio share of risky assets ($\mathrm{d}\omega_t = 0$), since the risk reduction due to the tax reform is exactly offset by the risk increase of the state-contingent rebate. In finance language: the state-contingent rebate is spanned by the risky asset, since it is proportional to the excess return $r_t(\theta_t) - \rho$. The individual's opportunity set is thus unchanged by the tax reform: the after-tax safe return is unchanged by construction. Therefore, the entire allocation is unchanged under the tax reform. The next Proposition demonstrates that a tax reform of replacing ex ante wealth taxes by capital income taxes is welfare neutral.

Proposition 3 (Equivalence of capital taxes with state-contingent lump-sum taxes)**.** *Starting from an initial tax system with arbitrary taxes on capital income and on wealth, presuming that all risk in capital income is aggregate risk, if the tax on capital income is raised, and the tax on wealth is lowered such that $\mathrm{d}\tau_t^A = -\rho \mathrm{d}\tau_t^I$, public goods provision is kept constant in every state ($\mathrm{d}G_{t+1} = 0$), and the government rebates any stochastic change in revenue by adjusting the state-contingent lump-sum tax such that $\mathrm{d}\mathcal{R}_{t+1} = 0$, and $\mathrm{d}T_t(\theta_t) \gtrless 0$ for any θ_t, then saving and portfolio choice are unaffected ($\mathrm{d}s_t = \mathrm{d}\omega_t = 0$) and social welfare in period t is exactly unchanged: $\frac{\mathrm{d}\mathcal{V}_t(a_t)}{\mathrm{d}\tau_t^I} = 0$.*

Proof See Appendix F.

[26] In contrast to Lemmas (3) and (5), this Lemma does *not* require portfolio choice to be interior. The reason is that the reform leaves next-period assets unchanged in every state, since the state-contingent tax exactly returns the risk that the higher tax on capital income takes away, see eq. (65) in the Appendix. The individual's opportunity set is unchanged, so that the pre-reform choices remain optimal whether or not the no-leverage constraint binds.

Proposition 3 establishes the neutrality results of Gordon (1985) and Kaplow (1994) in the dynamic Merton-Samuelson framework. Intuitively, the tax reform raises extra revenue in each state equal to $(r_t(\theta_t) - \rho)\omega_t s_t \mathrm{d}\tau_t^I$. By taking the expectation over all states, the government collects more taxes on the risk premium $\pi_t \omega_t s_t \mathrm{d}\tau_t^I$. The social value of these additional resources is zero, however, since there is also more risk in tax revenue. Since the tax on the risk premium is perfectly correlated with the return on risky assets, the social cost of risk in tax revenue is exactly equal to the revenue itself.

This is the mirror image of what happens in the private sector. On the one hand, individuals experience a reduction in their risk-exposure via the tax on capital income. On the other hand, they face an increase in their risk-exposure because the government returns all risky tax revenues back to individuals via state-contingent lump-sum transfers. The net income stream $\mathrm{E}_t[\mathcal{V}'_{t+1}(\cdot)(r_t(\theta_t) - \rho)]\omega_t s_t \mathrm{d}\tau_t^I = 0$ has zero value to the individual, which follows from the portfolio optimality condition (8). As a result, the government does *not* generate social value by taxing the risk premium. Importantly, the tax reform yields more tax revenue by taxing the risk-premium at higher rates even though individuals do not change their behaviour and social welfare does not change either. This is the reason that Gordon (1985) argues that the US can generate substantial revenue from taxing capital income without triggering large behavioural responses or welfare losses.

Under aggregate risk and state-contingent lump-sum taxes, taxes on capital income and (ex ante) taxes on wealth are therefore equivalent in welfare terms: replacing taxes on wealth by taxes on capital income produces exactly zero gain in social welfare.[27] Hence, taxes on capital income are no longer strictly preferred over taxes on wealth – as is the case with idiosyncratic risk or state-contingent public goods (see next section). At the same time, it can also not be concluded that taxes on wealth should be preferred over taxes on capital income.

4.6 Welfare effects tax reform – aggregate risk and state-contingent public goods

The comparison between the tax on capital income and on wealth becomes meaningful under aggregate risk only if the risk borne by the government is not directly borne by individuals as in the previous section. Therefore, this section analyses the policy experiment in which the government balances its budget by adjusting state-contingent public goods provision, as in Christiansen (1993), Schindler (2008) and Boadway and Spiritus (2025). Public goods enter utility directly, are not traded, and are therefore not spanned by traded assets.

In particular, the government adjusts taxes on capital income and wealth according to Lemma (1) (i.e. $\mathrm{d}\tau_t^A = -\rho \mathrm{d}\tau_t^I < 0$), *and* it adjusts public goods provision in period $t+1$ – when stochastic revenue from the period-t tax bases is collected – in each state θ_t such that eq. (12) holds under the reform (i.e. $\mathrm{d}\mathcal{R}_{t+1} = 0$, $\mathrm{d}G_{t+1} \gtrless 0$ for any θ_t), while the lump-sum tax is held constant ($\mathrm{d}T_t(\theta_t) = 0$ for all θ_t). To find the welfare effect of the reform, the change in expected utility is calculated under the tax reform *joint* with a state-contingent change in public goods provision to balance the budget.[28] This gives the next Proposition of the paper.

[27] It is straightforward to demonstrate that the equivalence between taxes on capital income and wealth remains if ex post wealth taxes are considered.

[28] If the same policy experiment is analysed for the case with idiosyncratic risk, i.e. transfer higher public

Proposition 4 (Superiority of taxes on capital income over taxes on wealth)**.** *Starting from an initial tax system with positive taxes on capital income and on wealth, presuming that all risk in capital income is aggregate risk, if the tax on capital income is raised, and the tax on wealth is lowered such that* $\mathrm{d}\tau_t^A = -\rho \mathrm{d}\tau_t^I$*, and the government rebates any stochastic change in resources by adjusting public goods provision in period* $t+1$ *such that* $\mathrm{d}\mathcal{R}_{t+1} = 0$*, and* $\mathrm{d}G_{t+1} \gtrless 0$ *for any* θ_t*, then social welfare changes in period* t *according to:*

$$\begin{aligned}\frac{\phi_t \mathrm{d}\mathcal{V}_t(a_t)/\mathrm{d}\tau_t^I}{u'(c_t)s_t} &= \left[\pi_t\omega_t - \frac{(\tau_t^I(\rho+\pi_t\omega_t)+\tau_t^A)}{1-\tau_t^I}\varepsilon_{s\tau^I} + \frac{\tau_t^I}{1-\tau_t^I}\pi_t\omega_t\varepsilon_{\omega\tau^I}\right. \\ &\quad \left. + \frac{\omega_t}{1-\tau_t^I}\mathrm{E}_t\left[\left(\frac{\Gamma'(G_{t+1})}{\mathrm{E}_t[u'(c_{t+1})]} - 1\right)(r_t(\theta_t)-\rho)\right]\right] \\ &= \frac{\omega_t(\pi_t - \pi_t^G)}{1-\tau_t^I}\frac{\mathrm{E}_t[\Gamma'(G_{t+1})]}{\mathrm{E}_t[u'(c_{t+1})]},\end{aligned} \tag{35}$$

where $\phi_t \equiv 1 + (1-\tau_t^I)\rho - \tau_t^A > 0$ *is the after-tax gross safe return factor and the elasticities follow from Lemma (3), which are imposed in the last term of the first line and in all subsequent lines. The* public *risk premium on equities in year* t *is defined as:*

$$\pi_t^G \equiv -\frac{\mathrm{cov}_t[r_t(\theta_t), \Gamma'(G_{t+1})]}{\mathrm{E}_t[\Gamma'(G_{t+1})]}. \tag{36}$$

Proof See Appendix G.

Note that the left-hand side captures the monetised value of the welfare gain as of period t. By multiplying by the after-tax safe return ϕ_t these gains are converted into period-$t+1$ units, which makes the left-hand side comparable with the corresponding expression in Proposition 1. This Proposition demonstrates that the welfare effects of the tax reform under aggregate risk are the same as the welfare effects under idiosyncratic risk, up to one term.

The first three terms in the first line of eq. (35) are the same as the welfare effects of the tax reform under idiosyncratic risk, cf. eq. (29). Their intuition is also the same. The government collects (stochastic) revenue from the risk premium as indicated by the first term. The tax reform has no effect on saving because the net safe return is not affected, like in all cases before, so the second term again cancels. The Domar-Musgrave effect in the third term represents the increase in risk-taking (with unit elasticity), just as with idiosyncratic risk. Larger risk-taking contributes to raising tax revenue.

The main difference with the idiosyncratic case is the fourth term on the second line. Under aggregate risk, taxing the risk premium generates stochastic tax revenue, which is returned in the form of state-contingent public goods. The fourth term is the social value of larger public goods provision net of the additional risk in public goods provision, since public goods are financed from stochastic tax revenue.

The third line of eq. (35) combines the first and the fourth term. The term $(\pi_t - \pi_t^G)$ measures the net social gain from shifting aggregate risk from private to public consumption. The main reason why the government likes to raise revenue from taxing the risk premium is that

revenue back in the form of larger public goods provision, the conclusions would remain unaffected. Intuitively, transferring higher, deterministic government revenue back in the form of transfers or public goods does not make a qualitative difference. See Appendix D.1.

it can diversify aggregate risk over private and public consumption, see Christiansen (1993), Schindler (2008) and Boadway and Spiritus (2025). $\pi_t^G \equiv -\frac{\text{cov}_t[r_t(\theta_t),\Gamma'(G_{t+1})]}{\text{E}_t[\Gamma'(G_{t+1})]}$ is the *public* risk-premium, which captures the monetary value of risk in public goods provision, since public goods provision fluctuates with stochastic tax revenue, which is linear in the return $r_t(\theta_t)$. The public risk premium is zero if the tax on capital income is zero and positive otherwise: since $\partial G_{t+1}/\partial r_t(\theta_t) = \tau_t^I \omega_t s_t$ and $\Gamma'' < 0$, public goods provision carries no return risk at $\tau_t^I = 0$, while $\Gamma'(G_{t+1})$ is decreasing in the realised return whenever $\tau_t^I > 0$. If the social gain of taxing the risk premium is larger than the social cost of more risk in public goods provision, i.e. $\pi_t > \pi_t^G$, then taxing the risk premium allows for better smoothing of aggregate risks over private and public consumption and social welfare increases (and vice versa). If aggregate risk is optimally smoothed over private and public goods, then $\pi_t = \pi_t^G$ – see Section 5 – and this term vanishes. The tax on capital income determines the degree of aggregate risk sharing. As long as the tax on capital income is below its optimal level derived in Section 5, the tax reform raises social welfare

$\frac{\text{E}_t[\Gamma'(G_{t+1})]}{\text{E}_t[u'(c_{t+1})]}$ captures whether public goods provision is sub-optimally high or not in the initial equilibrium before the tax perturbation. In particular, if public goods provision is sub-optimally low (high), then $\frac{\text{E}_t[\Gamma'(G_{t+1})]}{\text{E}_t[u'(c_{t+1})]} > 1 (< 1)$. An increase in expected tax revenue from taxing the risk premium allows for more public goods provision, which alleviates the under-provision of public goods (exacerbates the over-provision of public goods). Public goods provision is efficient if $\frac{\text{E}_t[\Gamma'(G_{t+1})]}{\text{E}_t[u'(c_{t+1})]} = 1$ – see also next section on optimal taxation. Whether public goods are under- or over-provided in the initial equilibrium is irrelevant for the *direction* of the welfare effect. In particular, if the initial tax system consists of a tax on wealth only ($\tau_t^I = 0$), then public goods provision carries no aggregate risk, so that $\pi_t^G = 0 < \pi_t$. Replacing taxes on wealth by taxes on capital income is then *always* welfare improving, irrespective of the level of public goods provision.

5 Optimal taxation

This Section analyses optimal taxes on capital income and wealth for the separate cases where risk is idiosyncratic or aggregate, where in the latter case state-contingent lump-sum taxes or public goods are considered. In doing so, this paper extends the analyses in Gordon (1985), Christiansen (1993), Kaplow (1994), Schindler (2008) and Boadway and Spiritus (2025) to an infinite-horizon Merton-Samuelson framework.

5.1 Optimal taxation with idiosyncratic risk

Proposition 5 gives optimal tax policy with idiosyncratic risk.

Proposition 5 (Optimal taxation and public good provision with idiosyncratic risk)**.** *If risk in capital income is idiosyncratic, portfolios satisfy the no-leverage constraint $\omega_t \leq 1$, the government optimises taxes on capital income and ex ante or ex post taxes on wealth, and it optimally provides public goods, then optimal policy is only defined if revenue from taxing the risk premium is insufficient to finance the wealth subsidy. Optimal policy then satisfies:*

1. *Public goods provision in each period t is distorted downwards:*

$$\Gamma'(G_t)/u'(c_t) > 1. \tag{37}$$

2. *Ex ante or ex post wealth subsidies in some or all periods t are insufficient to remove all intertemporal distortions ($-\tau_t^A = -\tau_t^P < \rho + \pi_t$), so that $\phi_t < 1 + \rho + \pi_t$.*

3. *The optimal tax on capital income in each period t is $\tau_t^I = 1$.*

4. *On a net basis, capital is taxed in any state in each period t as the government appropriates the entire risk premium $\pi_t = \mathrm{E}_t[r_t(\theta_t)] - \rho$.*

Proof See Appendix H.

Proposition 5 states that it is optimal to tax all capital incomes at 100 percent ($\tau_t^I = 1$) to fully insure all idiosyncratic risk. This is intuitive, since the government tries to obtain as much revenue from taxing the risk premium as possible. As a result, the Domar-Musgrave effect pushes individuals into a corner where their no-leverage constraint binds and all their wealth is invested in equities. The no-leverage constraint is essential here, and not a technical convenience: without it the equity position and the tax base would grow without bound as the tax on capital income approaches 100 percent, and the government's problem would have no solution at all. The revenue from taxing capital income is returned in the form of deterministic subsidies on wealth. However, this is insufficient to guarantee an efficient intertemporal allocation (i.e. such that $-\tau_t^A = -\tau_t^P = \rho + \pi_t$ and $\phi_t = 1 + \rho + \pi_t$) if the government needs (part of) the revenue from taxing the risk premium to finance public goods. Individuals have invested all their wealth in equities, which is fully insured ($\tau_t^I = 1$), and generates – in the aggregate – a safe rate of return equal to $\rho + \pi_t$. If not all revenue from taxing capital income can be returned to individuals in the form of wealth subsidies, the government programme becomes a second-best Ramsey problem. The combination of taxes on capital income and wealth then implies a positive net tax on the risk-free rate of return to capital and an under-provision of public goods – relative to the first-best rule. The government then trades off distortions in saving against distortions in providing public goods.[29]

It may come as a surprise that a first-best solution for optimal tax policy cannot be obtained, since this is what Boadway and Spiritus (2025) have obtained. To see why, suppose that the revenue from taxing the risk premium is sufficiently large to finance wealth subsidies to ensure intertemporal efficiency, i.e. $-\tau_t^A = -\tau_t^P = \rho + \pi_t$ and $\phi_t = 1 + \rho + \pi_t$. Suppose furthermore that exogenous transfers are sufficiently large for the government to exactly provide the efficient level of public goods. Then, the individual receives the social return $1 + \rho + \pi_t$ on its saving, its consumption is deterministic (full insurance), its portfolio is fully invested in the high-return asset, and public goods provision follows the first-best Samuelson rule ($\Gamma'(G_t)/u'(c_t) = 1$). Still,

[29] The optimal tax on capital income of 100 percent depends on the assumption that the return on the risky asset is exogenous. Buchholz and Konrad (2014) argue that this neutrality breaks down with limited liability, moral hazard or adverse selection in financial markets. Hence, full insurance through the tax system distorts incentives to supply effort, to select good projects and to bear entrepreneurial risk, and a 100 percent tax on capital income ceases to be optimal. Since this paper abstracts from incentive effects, as does the two-period literature to which it relates, it should be regarded as the upper bound.

this is not an optimal policy solution because the individual Euler equation is inconsistent with the government's Euler equation – which follows from the optimal path of government debt – see Appendix H:

$$\underbrace{\frac{u'(c_t)}{u'(c_{t+1})} = \frac{1+\rho}{1+\beta}}_{\text{government Euler}} \quad \text{vs.} \quad \underbrace{\frac{u'(c_t)}{u'(c_{t+1})} = \frac{1+\rho+\pi_t}{1+\beta}}_{\text{private Euler}}. \tag{38}$$

Indeed, the government and private sector Euler equations cannot hold simultaneously unless the risk premium is zero: $\pi_t = 0$. Hence, for a positive risk premium $\pi_t > 0$, the optimality conditions for taxes and public goods have *no* solution with a zero intertemporal wedge. The fundamental issue here is that, at $\tau^I = 1$, the private sector transforms resources over time at the deterministic pooled return $1+\rho+\pi_t$, while the government transforms resources over time at the safe return $1+\rho$. Hence, there are two safe saving technologies with different returns, which cannot both be priced consistently in one intertemporal optimum. Consequently, the joint allocation of the private and the public sector is not arbitrage-free once the government can shift resources between the two saving technologies.[30]

The source of the inconsistency is the no-leverage constraint, and it went unnoticed in the literature because that constraint is not imposed there, see e.g. Boadway and Spiritus (2025). Without imposing a maximum on the portfolio share on equities, the Domar-Musgrave effect makes the equity position and the tax base grow without bound as τ_t^I approaches one. Hence, revenue from taxing the risk premium is unbounded at no utility cost and no optimum exists for that reason. Once the constraint is imposed, the optimal-tax problem is well-defined, but the corner solution in portfolio choice causes the intertemporal inconsistency. All wealth is then invested in equity, so that the marginal unit of saving earns the pooled return $\rho+\pi_t$, while the government continues to allocate resources over time at the safe return ρ. The benchmark result of Boadway and Spiritus (2025) – that the normal return to saving should remain untaxed – can therefore not be established as an optimum condition, because it is inconsistent with the intertemporal optimality conditions of the government and the private sector.[31]

This methodological problem can only be solved by departing from the general structure of the model. Two modifications could possibly address the issue. First, the partial-equilibrium assumption can be dropped and general equilibrium in asset markets can be introduced. If the safe asset is in zero net supply between households and the government (or in a closed capital market), the safe rate is endogenous rather than a second, exogenous saving technology, see also Angeletos and Calvet (2006) and Angeletos (2007). Once the government insures idiosyncratic risk through the tax system, the premium disappears and the safe rate rises towards the pooled return, so that any arbitrage opportunity between the two saving technologies is priced away. As long as one assumes CRRA preferences and linear individual technologies, exact aggregation remains possible and the tractability of the Merton-Samuelson framework survives, as in, for

[30] It might be conjectured that this problem is an artefact of treating the deterministic lump-sum tax T_t as exogenous, so that the government is forced to leave a positive intertemporal wedge on saving. Appendix H.1 shows that it is not: adding T_t to the government's instrument set does not restore the first best. Rather, allowing for lump-sum taxes would remove the only (second-best) case in which an optimum is well-defined, so that the conditions for optimal tax policy would no longer have a solution.

[31] Theorem 2 of Boadway and Spiritus (2025) contains a cross-base term $-t^e \int (\mathrm{E}[\bar{y}_t^{ecs}] - \chi^{cs})$, which is the compensated effect of the normal-return tax on the excess-return tax base. The correction of the wealth tax from $-\rho$ to $-(\rho+\pi_t)$ corresponds to this term in the tax optimum with only idiosyncratic risk.

example, Angeletos (2007).[32] Second, one could give up the linearity of the risky technology at the individual level, and model risky returns as arising from private projects with decreasing returns to scale, as in the literature on entrepreneurship and type-dependent returns in Gerritsen et al. (2025). The safe asset is then the only linear saving technology, the excess return on the private project is a rent, and no second linear technology exists whose return the government's Euler equation could contradict. This modification, however, sacrifices exact aggregation. Both extensions are left for future research.

5.2 Optimal taxation with aggregate risk and state-contingent lump-sum taxes

Proposition 6 gives optimal tax policy with aggregate risk and state-contingent lump-sum taxes.

Proposition 6 (Optimal taxation with aggregate risk and lump-sum taxes)**.** *If risk in capital income is aggregate risk, and the government optimises taxes on capital income, ex ante or ex post taxes on wealth, state-contingent lump-sum taxes, public goods provision and government debt, then optimal policy decentralises the first best allocation:*

1. *Public goods provision in each period t satisfies the Samuelson rule for each state θ_t so that the risk-adjusted Samuelson rule holds in expectation:*

$$\Gamma'(G_t(\theta_t)) = \frac{1+\rho}{1+\beta} u'(c_{t+1}(\theta_t)),\ \forall \theta_t, \quad \frac{\mathrm{E}_t[\Gamma'(G_t(\theta_t))]}{u'(c_t)} = 1. \tag{39}$$

2. *Ex ante or ex post wealth taxes in each period t ensure intertemporal efficiency in saving ($\tau_t^A = -\tau_t^I \rho$ or $\tau_t^P = -\frac{\tau_t^I \rho}{1+(1-\tau_t^I)\rho}$), so that the after-tax safe return is undistorted: $\phi_t = 1+\rho$.*

3. *The optimal tax on capital income in each period t is indeterminate: every $\tau_t^I \in [0,1)$, combined with the optimal wealth tax, and correspondingly adjusted state-contingent lump-sum taxes, implements the same first-best allocation.*

4. *On a net basis, capital is not taxed in any state in each period t, since total revenue from taxes on capital income and wealth, $\tau_t^I(r_t(\theta_t)-\rho)\omega_t s_t$, which is positive in expectation, is returned to individuals through state-contingent lump-sum taxes.*

Proof See Appendix I.

This proposition is the optimal-tax counterpart of the neutrality results of Gordon (1985), Kaplow (1994) and Hamilton (1987), which are derived for given tax rates. Moreover, it generalises the results in Boadway and Spiritus (2021), in the working-paper version of Boadway and Spiritus (2025), from two-period to infinite-horizon models. With a complete set of state-contingent lump-sum taxes the government reaches the first best. The optimal tax on capital income is indeterminate in the full optimum. That the indeterminacy arises is perhaps not

[32] Panousi (2010) and Panousi and Reis (2012) study optimal linear capital taxes in this class of models, but do not compare taxes on capital income with taxes on wealth.

surprising, since taxes on capital income and on wealth are equivalent in welfare terms once all revenue risk is returned state by state, as was already recognised by Gordon (1985), Kaplow (1994) and Hamilton (1987).

Relative to this literature, the contribution of Proposition 6 is threefold. First, the indeterminacy of the tax on capital income is *derived* from the first-order conditions of the government's problem, rather than asserted as a neutrality property at given tax rates. Second, Proposition 6 pins down the optimal structure of taxes on wealth and capital income, $\tau_t^A = -\tau_t^I \rho$ (or $\tau_t^P = -\tau_t^I \rho/(1 + (1 - \tau_t^I)\rho)$ for the ex post wealth tax), on which the neutrality literature is silent.

Third, it derives the state-by-state Samuelson rule (39) jointly with the government's Euler equation for public debt (138). In doing so, the infinite-horizon model can answer the question whether allowing for government debt is helpful to smooth out aggregate risks over time.[33] Proposition 7 answers it in the negative: government debt is *not* used to smooth out aggregate risk over time. Intuitively, government debt is not state contingent and therefore cannot provide direct insurance against aggregate risk. Moreover, the government has no advantage over individuals in allocating resources over time. Individuals trade the same safe asset at the same rate as the government. Since the model has no capital market frictions, any debt-financed reshuffling of non-distortionary taxes over time is undone one-for-one by individuals and Ricardian equivalence holds. Alternatively, in a representative-agent setting, aggregate risk cannot be diversified across time, only across individuals. In a representative-agent economy, future periods are populated by the same individuals, so there is no counterparty. Allowing for overlapping generations or capital market frictions (e.g. borrowing constraints) could generate a meaningful role for public debt in smoothing out aggregate risks.

The results of Hamilton (1987) are closest to this Proposition. His 'equal-welfare locus' $t_w = rt_y$ – in his notation, where t_w is the wealth tax, t_y the tax on capital income and r the safe rate of return – coincides with $\tau_t^A = -\tau_t^I \rho$ derived here. He derives this as an indifference result rather than an optimal policy rule.[34]

5.3 Optimal taxation with aggregate risk and state-contingent public goods

Proposition 7 gives optimal tax policy with aggregate risk and state-contingent public goods.

Proposition 7 (Optimal taxation and public good provision with aggregate risk)**.** *If risk in capital income is aggregate risk, and the government optimises taxes on capital income, ex ante or ex post taxes on wealth, state-contingent public good provision and government debt, then optimal policy satisfies:*

1. *Public goods provision is stochastic in each period t so that the risk-adjusted Samuelson rule holds in expectation*

$$\frac{\mathrm{E}[\Gamma'(G_t)]}{u'(c_t)} = 1. \tag{40}$$

[33] Boadway and Spiritus (2021) assume that the government cannot pool aggregate risk over time by borrowing, and therefore leave the question open.

[34] Related is also Christiansen (1993), who notes that a first-best outcome requires state-contingent lump-sum taxes, but characterises the second best in which taxes are restricted to be state-independent, rather than deriving the optimal tax system in the first-best case.

2. *Ex ante or ex post wealth taxes in each period t ensure intertemporal efficiency in saving ($\tau_t^A = -\tau_t^I \rho$ or $\tau_t^P = -\frac{\tau_t^I \rho}{1+(1-\tau_t^I)\rho}$), so that the after-tax safe return is undistorted: $\phi_t = 1 + \rho$.*

3. *The optimal tax on capital income in each period t is strictly positive but below 100 percent: $\tau_t^I \in (0, 1)$. The optimal tax rate on capital income is determined by optimal risk-sharing of aggregate risk over private and public goods ($\pi_t = \pi_t^G$):*

$$\frac{\mathrm{cov}_t[r_t(\theta_t), \mathcal{V}'_{t+1}(\cdot)]}{\mathrm{E}_t[\mathcal{V}'_{t+1}(\cdot)]} = \frac{\mathrm{cov}_t[r_t(\theta_t), \Gamma'(G_{t+1})]}{\mathrm{E}_t[\Gamma'(G_{t+1})]}. \tag{41}$$

4. *On a net basis, capital is taxed on average in each period t, because the risk-premium is taxed.*

Proof See Appendix J.

This proposition again demonstrates that the insights derived from two-period frameworks in Christiansen (1993), Schindler (2008) and Boadway and Spiritus (2025) fully generalise to the infinite-horizon Merton-Samuelson framework. In contrast to Proposition 3, taxing the risk premium now does generate social value. Public goods change the state-contingency of the allocation. State-contingent public goods are untraded and not spanned by bonds or equities. Therefore, individuals cannot undo the state-contingent variation in public goods through portfolio adjustments, so that the government shares in aggregate risk. The risk-adjusted Samuelson rule implies that aggregate risk is optimally diversified over private and public goods.

Implementation of the optimal risk-sharing arrangement between the public and private sector requires a positive tax on capital income. Optimality requires that the government values its equity exposure – which it holds through the stochastic revenue from taxing the risk premium – at the same risk-adjusted price as individuals do, as in Christiansen (1993). The optimal tax on capital income is positive for the reasons given in Section 4. The public risk premium π_t^G equals zero at $\tau_t^I = 0$ and increases in the tax on capital income. The government keeps raising the tax on capital income until its own risk premium equals the private risk premium. The optimal tax on capital income is larger if there is more (less) risk aversion in private consumption (public goods).

A positive subsidy on wealth eliminates any intertemporal distortion arising from the tax on capital income, so that only the risk-premium is taxed. This confirms Schindler (2008) and Boadway and Spiritus (2025), who show that there is no trade-off between efficiency in intertemporal consumption and the efficient allocation of risk, provided that the excess return is taxed at its own tax rate. The optimal tax implementation with taxes on capital income and wealth in this paper is economically equivalent.

Proposition 7 once more demonstrates that government debt cannot be used for smoothing aggregate risks over time. See the discussion of the previous section.

6 Optimal time-invariant taxation

Most real-world tax systems employ time-invariant taxes. It is therefore instructive to analyse the cases in which tax rates on capital income are constant over time. However, with aggregate risk, there still needs to be an instrument – lump-sum taxes or public goods – that ensures the government budget constraint holds for each state. Hence, these instruments cannot be made time-invariant by construction.

This Section thus imposes the constraint that taxes on capital income and wealth are constant over time (and not state-contingent): $\tau_t^I = \tau^I$ and $\tau_t^A = \tau^A$ (or $\tau_t^P = \tau^P$) for all t and θ_t. Since the safe rate of return ρ is constant, constant tax rates ensure that the after-tax safe return factor is also constant over time: $\phi_t = \phi \equiv 1+(1-\tau^I)\rho-\tau^A$, or $\phi = (1-\tau^P)(1+(1-\tau^I)\rho)$ for the ex post wealth tax.

Because a common tax rate applies in every period, a change in the tax rate changes behaviour in all time periods simultaneously. Therefore, the behavioural response of savings s at date s is the *sum* of its partial responses to the period-specific rates at all dates t, $\sum_{t=0}^{\infty} \partial s_s/\partial \phi_t$ and $\sum_{t=0}^{\infty} \partial s_s/\partial \tau_t^I\big|_{\phi_t}$, and analogously for the net equity exposure x_s. The following three Propositions mirror Propositions 5, 6 and 7 for time-invariant tax rates.

Proposition 8 (Time-invariant taxes with idiosyncratic risk)**.** *If risk in capital income is idiosyncratic, portfolios satisfy the no-leverage constraint $\omega_t \leq 1$, the government optimises time-invariant taxes on capital income and ex ante or ex post taxes on wealth, and it optimally provides public goods, then optimal policy is only defined if revenue from taxing the risk premium is insufficient to finance a wealth subsidy equal to $\rho + \pi$. Optimal policy then satisfies:*

1. *Public goods provision is distorted downwards in discounted-average form:*

$$\sum_{t=0}^{\infty} \frac{s_t u'(c_{t+1})}{(1+\beta)^{t+1}} \left[\frac{\Gamma'(G_{t+1})}{u'(c_{t+1})} - 1 \right] > 0. \tag{42}$$

2. *Ex ante or ex post wealth subsidies are insufficient to remove all intertemporal distortions ($\tau^A = \tau^P > -(\rho+\pi)$), so that $\phi < 1 + \rho + \pi$.*

3. *The optimal tax on capital income is $\tau^I = 1$, so that portfolios are fully invested in risky assets in every period ($\omega_t = 1$ for all t).*

4. *On a net basis, capital is taxed in every period t, since the wealth subsidy falls short of the risk premium $\pi = \mathrm{E}_t[r_t(\theta_t)] - \rho$ that the government appropriates.*

Proposition 8 mirrors Proposition 5. As there, the no-leverage constraint binds at the optimum, so that the marginal unit of saving earns the pooled return $\rho+\pi$ while the government prices resources at the safe return ρ. The restriction to time-invariant tax rates does *not* restore a first-best solution under idiosyncratic risk. This is not obvious. The inconsistency in Proposition 5 arises because the per-period Samuelson rule and the two Euler equations cannot hold simultaneously, and the restriction to constant tax rates removes exactly that requirement: with a single tax on capital income and a single tax on wealth, only the *discounted-average*

Samuelson rule (42) has to hold. However, public goods provision and public debt remain unrestricted, so their first-order conditions continue to hold period by period. They imply that the marginal utility of public goods grows at the gross rate $(1+\beta)/(1+\rho)$: the government prices resources over time at the safe return. At $\tau^I = 1$ consumption is deterministic and the individual's Euler equation implies that the marginal utility of private consumption grows at $(1+\beta)/\phi$: the private sector prices resources over time at the after-tax return ϕ. Dividing the two gives the ratio of the marginal utility of public goods to the marginal utility of private consumption:

$$\frac{\Gamma'(G_{t+1})}{u'(c_{t+1})} = \frac{\Gamma'(G_1)}{u'(c_1)} \left(\frac{\phi}{1+\rho}\right)^t. \tag{43}$$

Also in case the tax rates are constant, the wedge between the two saving technologies does not cancel: it *compounds* at the gross rate $\phi/(1+\rho)$.

The first-best solution $\phi = 1+\rho+\pi$ does not work in two, distinct ways. First, eq. (43) then grows without bound, so that public goods provision is driven to zero relative to private consumption. The per-period Samuelson rule is violated by an ever-increasing factor. This contradicts optimality principles. Second, the discounted-average Samuelson rule has an interior solution only if the present value of saving discounted at the *government's* rate converges, i.e. $\sum_t s_t(1+\rho)^{-t} < \infty$. This is strictly stronger than the transversality condition of the individual, which only requires convergence at the *private* rate $\phi = 1+\rho+\pi$, and it fails whenever saving grows faster than $1+\rho$ – as it does, for example, under logarithmic utility. In that case no interior optimum exists at all: the government borrows at $1+\rho$ while the economy transforms resources at $1+\rho+\pi$, which yields arbitrage that is never exhausted. The wealth subsidy that removes all intertemporal distortions is therefore not part of an optimum, exactly as in Proposition 5.

The compounding in eq. (43) stops if and only if $\phi = 1+\rho$, i.e. if the wealth subsidy equals the safe return rather than the pooled return: $\tau^A = -\rho$ at $\tau^I = 1$. The government and the private sector then price resources over time identically, the ratio (43) is constant, and public goods are *under*-provided by the same factor in every period. Any optimum must therefore satisfy $\phi \leq 1+\rho$, so that the intertemporal wedge is at least as large as the risk premium, $\mathcal{W} \geq \pi$, with equality when the government's budget is exactly met at $\phi = 1+\rho$. Optimal policy under idiosyncratic risk with time-invariant taxes is thus a tax on capital income joint with an allowance for the safe rate of return – as in the two cases with aggregate risk in Section 5 – but, unlike those cases, with a strictly positive intertemporal wedge and an under-provision of public goods.

Proposition 9 (Time-invariant taxes with aggregate risk and lump-sum taxes)**.** *If risk in capital income is aggregate risk, and the government optimises time-invariant taxes on capital income and ex ante or ex post taxes on wealth, state-contingent lump-sum taxes, public goods provision and government debt, then optimal policy decentralises the first best allocation:*

1. *Public goods provision in each period t satisfies the Samuelson rule for each state θ_t, so that the risk-adjusted Samuelson rule holds in expectation, eq. (39).*

2. *Ex ante or ex post wealth taxes ensure intertemporal efficiency in saving ($\tau^A = -\tau^I\rho$ or*

$\tau^P = -\frac{\tau^I \rho}{1+(1-\tau^I)\rho}$*), so that the after-tax safe return is undistorted:* $\phi = 1 + \rho$.

3. *The optimal tax on capital income is indeterminate: every* $\tau^I \in [0, 1)$*, combined with the optimal wealth tax and correspondingly adjusted state-contingent lump-sum taxes, implements the same first-best allocation.*

4. *On a net basis, capital is not taxed in any state in any period, since total revenue from taxes on capital income and wealth, which is positive in expectation, is returned to individuals through the state-contingent lump-sum taxes.*

Proposition 9 shows that imposing time-invariant taxes is entirely harmless. The first best of Proposition 6 is still decentralised, the Samuelson rule still holds state by state, the after-tax safe return is still undistorted and the tax on capital income is still indeterminate on the whole interval $\tau^I \in [0, 1)$. The reason is simple. With time-varying taxes, the tax on capital income has no impact on the allocation in *any* period. The state-contingent lump-sum taxes exactly return all state-dependent revenue state by state. Hence, if an instrument is redundant at every date, it remains redundant when it is restricted to be constant over time. The optimal wealth tax $\tau^A = -\tau^I \rho$ is then automatically constant as well, since the safe return ρ is constant.

Proposition 10 (Time-invariant taxes with aggregate risk and public goods)**.** *If risk in capital income is aggregate risk, lump-sum taxes are fixed, and the government optimises time-invariant taxes on capital income and ex ante or ex post taxes on wealth, state-contingent public goods provision and government debt, then optimal policy satisfies:*

1. *Public goods provision is stochastic in every period* t*. If the risk-allocation condition holds period by period,* $\mathrm{E}_t[\Gamma'(G_t(\theta_t))(r_t(\theta_t) - \rho)] = 0$ *for all* t *– as is the case whenever the optimal tax rates of Proposition 7 are themselves constant over time – then the risk-adjusted Samuelson rule holds in discounted-average form:*

$$\sum_{t=0}^{\infty} \frac{1}{(1+\beta)^t} \mathrm{E}_0 \left[\left(\mathrm{E}_t[\Gamma'(G_t(\theta_t))] - u'(c_t) \right) s_t \right] = 0. \tag{44}$$

2. *In that case, ex ante or ex post wealth taxes ensure intertemporal efficiency in saving (*$\tau^A = -\tau^I \rho$ *or* $\tau^P = -\frac{\tau^I \rho}{1+(1-\tau^I)\rho}$*), so that the after-tax safe return is undistorted:* $\phi = 1 + \rho$*. Otherwise, the after-tax safe return solves*

$$\begin{aligned} &\sum_{t=0}^{\infty} \frac{1}{(1+\beta)^{t+1}} \mathrm{E}_0 \left[\left(\frac{(1+\beta)u'(c_t)}{\phi} - \frac{1+\beta}{1+\rho} \mathrm{E}_t[\Gamma'(G_t(\theta_t))] \right) s_t \right] \\ &+ \sum_{t=0}^{\infty} \sum_{s=0}^{\infty} \frac{1}{(1+\beta)^{s+1}} \mathrm{E}_0 \left[\mu_{s+1} \left((1+\rho-\phi) \frac{\partial s_s}{\partial \phi_t} + \tau^I (r_s(\theta_s) - \rho) \frac{\partial x_s}{\partial \phi_t} \right) \right] = 0, \end{aligned} \tag{45}$$

   where $\mu_{s+1} = \Gamma'(G_{s+1}(\theta_{s+1}))$*, and generally* $\phi \neq 1 + \rho$.

3. *The optimal tax on capital income is strictly positive,* $\tau^I > 0$*, and satisfies the risk-*

allocation condition in discounted-average form:

$$\sum_{t=0}^{\infty} \frac{1}{(1+\beta)^t} \mathrm{E}_0 \left[x_t \, \mathrm{E}_t[\Gamma'(G_t(\theta_t))(r_t(\theta_t) - \rho)] \right] = 0. \tag{46}$$

4. *On a net basis, capital is taxed on average in every period t, because the risk-premium is taxed.*

Proof See Appendix K.

Proposition 10 shows that constraining the capital tax rates to be time-invariant is *not* harmless if the government returns revenue through state-contingent public goods. The economic intuition of Proposition 7 carries over: the tax on capital income remains strictly positive and interior. It is still determined by equating the public risk premium to the private risk premium. However, that equality now holds for the discounted average, see eq. (46). The same is true of the risk-adjusted Samuelson rule, eq. (44). Since each period enters the discounted sum weighted by its own equity exposure x_t, the optimal tax on capital income is an exposure-weighted average of the time-dependent optimal tax rates. Hence, risk-sharing between private and public consumption is efficient only on average and not state by state. Consequently, the restriction of constant tax rates results in welfare losses. In particular, the government bears too little aggregate risk in periods in which the private risk premium is relatively high ($\pi_t > \pi_t^G$) and bears too much aggregate risk in periods in which it is relatively low.

The most consequential difference appears in part 2 of Proposition 10. Under time-varying rates, the wealth tax could always be set so as to remove the intertemporal wedge, $\phi_t = 1 + \rho$. With a single after-tax safe return that is only true if the per-period risk-allocation conditions all hold. If they do not, a change in ϕ shifts equity exposure in periods in which the government's valuation of exposure is not zero. In that case, the wealth tax has a second task: besides ensuring intertemporal efficiency, also reallocating aggregate risk across periods. The optimum then solves eq. (45) and generally features $\phi \neq 1 + \rho$. The direction is intuitive. If a higher after-tax safe return raises equity exposure by more in periods in which the government is under-exposed to aggregate risk ($\pi_s > \pi_s^G$) than in periods in which it is over-exposed, then the wealth subsidy is larger than the one that merely offsets the tax on capital income ($\phi > 1 + \rho$), and vice versa. Hence, the clean dichotomy of Section 5 – the tax on capital income allocates risk, the tax on wealth prices consumption over time – is not a property of the model but a consequence of having one instrument per period. Not taxing the safe rate of return survives the restriction to time-invariant rates only in a stationary environment, in which the unrestricted optimum is itself constant over time.

To summarise, Propositions 8–10 leave the main conclusions of Section 5 largely intact. The optimal tax on capital income is 100 percent if risk is idiosyncratic, indeterminate if risk is aggregate and the government returns its revenue through state-contingent lump-sum taxes, and positive but below 100 percent if it returns its revenue through state-contingent public goods. The main role of wealth taxes remains to ensure intertemporal efficiency, though not always. Optimal policy continues generally combine a tax on capital income with a wealth subsidy (or an allowance for the safe rate of return). However, as soon as the government has

fewer instruments than margins, the wealth tax is used as well to smooth aggregate risk. In that case, the intertemporal allocation and provision of public goods are no longer efficient.

7 Conclusion

Should we tax capital income or wealth? This paper's answer is: we should tax capital income. This conclusion is derived by analysing taxes on capital income and wealth in the canonical infinite-horizon Merton-Samuelson portfolio model with safe and risky assets where there may either be idiosyncratic risk, aggregate risk with state-contingent lump-sum taxes or aggregate risk with state-contingent public goods.

This paper's methodological innovations are that tax reforms are analysed that keep expected utility fixed such that the net after-tax safe rate and utility remain constant. Moreover, Roy's identity has been derived from the Bellman equation. Tax reforms are found to be weakly welfare-improving, because the tax burden is shifted from the normal return to the risk premium. The latter is desirable if risks are idiosyncratic or allow for smoothing of aggregate risks to public goods. It is neutral if the higher tax on the risk-premium is returned via state-contingent lump-sum transfers. In all cases, tax revenues are higher. Optimal taxes on capital income are 100 percent, positive or indeterminate if risks are idiosyncratic, aggregate with state-contingent public goods or aggregate with state-contingent transfers. Subsidies on wealth ensure intertemporal efficiency. Tax implementations with a safe rate-of-return allowance or with a tax on the risk-premium only are equivalent.

This paper thus demonstrates that the recent literature with a strong focus on wealth taxes may lead policy makers astray. Indeed, this paper provides support to the traditionally strong preference of public finance scholars to tax capital income, and not wealth. Therefore, one may need to resort to other reasons why taxing wealth over taxing capital income may be desirable. Capital market imperfections may result in pre-existing distortions which might be alleviated better with wealth taxes, although the literature has not settled down on this issue (Guvenen et al., 2023; Boar and Midrigan, 2023; Guvenen et al., 2024). Taxing idiosyncratic capital returns might also distort incentives for effort, project selection and entrepreneurship once capital markets are imperfect, from which this paper abstracts, see Buchholz and Konrad (2014). Moreover, lock-in effects in taxing capital gains may also make wealth taxes desirable as a second-best instrument to tax capital if taxes on capital gains are hard to implement (Saez et al., 2021; Schelderup and Zoutman, 2024). It is therefore an interesting avenue for future research to extend the current analysis with capital market imperfections, entrepreneurship and capital gains.

A Proof Bellman recursion

This appendix derives the Bellman recursion in eq. (6) for the infinite-horizon model using the principle of optimality.

A feasible plan from period t onwards is a collection of history-measurable consumption and portfolio policies, $\{c_s(\cdot), \omega_s(\cdot)\}_{s\geq t}$, that satisfy the budget identities $c_s = y_s + a_s - s_s$ and

$a_{s+1} = s_s R_s - T_s(\theta_s)$ for all $s \geq t$, together with the no-Ponzi condition on assets. The value function is defined directly from the sequence problem:

$$\mathcal{V}_t(a_t) \equiv \max_{\{c_s(\cdot),\omega_s(\cdot)\}_{s=t}^{\infty}} \mathrm{E}_t \sum_{s=t}^{\infty} \frac{u(c_s) + \Gamma(G_s)}{(1+\beta)^{s-t}}. \tag{47}$$

The sum converges and the value function is finite by the assumptions imposed on utility in the main text, and the value function is the limit of its finite-horizon truncations, see Alvarez and Stokey (1998).[35]

Separate the period-t terms in eq. (47) from $s = t+1, \cdots, \infty$, and apply the law of iterated expectations, $\mathrm{E}_t[\cdot] = \mathrm{E}_t[\mathrm{E}_{t+1}[\cdot]]$:

$$\begin{aligned}
\mathcal{V}_t(a_t) &= \max_{\{c_s(\cdot),\omega_s(\cdot)\}_{s=t}^{\infty}} u(c_t) + \mathrm{E}_t[\Gamma(G_t)] + \frac{1}{(1+\beta)} \mathrm{E}_t \left[\mathrm{E}_{t+1} \sum_{s=t+1}^{\infty} \frac{u(c_s) + \Gamma(G_s)}{(1+\beta)^{s-t-1}} \right] \\
&= \max_{c_t,\omega_t} u(c_t) + \mathrm{E}_t[\Gamma(G_t)] + \frac{1}{(1+\beta)} \mathrm{E}_t \left[\max_{\{c_s(\cdot),\omega_s(\cdot)\}_{s=t+1}^{\infty}} \mathrm{E}_{t+1} \sum_{s=t+1}^{\infty} \frac{u(c_s) + \Gamma(G_s)}{(1+\beta)^{s-t-1}} \right] \\
&= \max_{c_t,\omega_t} u(c_t) + \mathrm{E}_t[\Gamma(G_t)] + \frac{1}{(1+\beta)} \mathrm{E}_t[\mathcal{V}_{t+1}((y_t + a_t - c_t)R_t - T_t(\theta_t))].
\end{aligned} \tag{48}$$

The second equality interchanges the maximisation over continuation plans with the conditional expectation over the current state θ_t. This interchange is the principle of optimality, and it is valid for two reasons. First, continuation plans are measurable functions of the history, so that they may be chosen separately for every realisation of θ_t. Second, the feasible set of continuation plans depends on the past only through next-period assets $a_{t+1} = (y_t + a_t - c_t)R_t - T_t(\theta_t)$. Consequently, choosing the continuation plan optimally state-by-state is feasible, and no feasible plan can improve upon it. The third equality substitutes the definition of $\mathcal{V}_{t+1}$ in eq. (47). This establishes the Bellman recursion (6) for every t.

Given the no-Ponzi condition on assets, optimality additionally requires the transversality condition:

$$\lim_{s\to\infty} \frac{1}{(1+\beta)^s} \mathrm{E}_t[u'(c_{t+s}) a_{t+s}] = 0, \tag{49}$$

where $u'(c_{t+s}) = \mathcal{V}'_{t+s}(a_{t+s})$ by the envelope condition in Lemma (2). Together with the first-order conditions below and the no-Ponzi condition, the transversality condition is sufficient for optimality, given strict concavity of the objective.

Maximisation of the right-hand side of the Bellman recursion with respect to c_t and ω_t – using $\partial R_t / \partial \omega_t = (1 - \tau_t^I)(r_t(\theta_t) - \rho)$ and the fact that each individual takes the non-individualised lump-sum tax $T_t(\theta_t)$ as given – yields for any t:

$$u'(c_t) = \frac{1}{(1+\beta)} \mathrm{E}_t[\mathcal{V}'_{t+1}(\cdot) R_t], \tag{50}$$

$$\mathrm{E}_t[\mathcal{V}'_{t+1}(\cdot)(y_t + a_t - c_t)(1 - \tau_t^I)(r_t(\theta_t) - \rho)] = 0, \tag{51}$$

[35] Existence and measurability of optimal policies follow from standard arguments, see Bertsekas and Shreve (1978). Under aggregate risk with serially correlated states, the value function also depends on the history θ^{t-1} through the conditional density $f(\theta_t \mid \theta^{t-1})$; this dependence is suppressed in the notation, in line with the notational conventions in the main text.

which are eqs. (7) and (8) in the main text. The solution of these first-order conditions yields optimal consumption c_t^* and optimal portfolio choice ω_t^* for any t.

B Proof Lemma 2: dynamic stochastic Roy's identity

Part (i). The period-t Bellman recursion is

$$\mathcal{V}_t(a_t) \equiv \max_{c_t,\omega_t} u(c_t) + \mathrm{E}_t[\Gamma(G_t(\theta_t))] + \frac{1}{(1+\beta)}\mathrm{E}_t[\mathcal{V}_{t+1}((y_t + a_t - c_t)R_t - T_t(\theta_t))], \tag{52}$$

with $R_t = 1 + (1-\tau_t^I)\rho - \tau_t^A + (1-\tau_t^I)(r_t(\theta_t) - \rho)\omega_t = \phi_t + (1-\tau_t^I)(r_t(\theta_t) - \rho)\omega_t$. By the envelope theorem, the derivatives of $\mathcal{V}_t$ with respect to a_t and the policy variables are the partial derivatives of the maximand at the optimal (c_t, ω_t):

$$\begin{aligned}
\frac{\partial \mathcal{V}_t(a_t)}{\partial a_t} &= \frac{1}{(1+\beta)}\mathrm{E}_t[\mathcal{V}'_{t+1}(\cdot)R_t] = u'(c_t),\\
\frac{\partial \mathcal{V}_t(a_t)}{\partial \tau_t^I} &= -\frac{1}{(1+\beta)}\mathrm{E}_t[\mathcal{V}'_{t+1}(\cdot)(y_t + a_t - c_t)(\rho(1-\omega_t) + r_t(\theta_t)\omega_t)],\\
\frac{\partial \mathcal{V}_t(a_t)}{\partial \tau_t^A} &= -\frac{1}{(1+\beta)}\mathrm{E}_t[\mathcal{V}'_{t+1}(\cdot)(y_t + a_t - c_t)],\\
\frac{\partial \mathcal{V}_t(a_t)}{\partial \phi_t} &= \frac{1}{(1+\beta)}\mathrm{E}_t[\mathcal{V}'_{t+1}(\cdot)(y_t + a_t - c_t)],\\
\left.\frac{\partial \mathcal{V}_t(a_t)}{\partial \tau_t^I}\right|_{\phi_t} &= -\frac{1}{(1+\beta)}\mathrm{E}_t[\mathcal{V}'_{t+1}(\cdot)(y_t + a_t - c_t)(r_t(\theta_t) - \rho)\omega_t],
\end{aligned}$$

where the second equality in the first line is the first-order condition for c_t, and the last two lines use the reparametrisation $R_t = \phi_t + (1-\tau_t^I)(r_t(\theta_t) - \rho)\omega_t$: $\partial R_t/\partial \phi_t = 1$ and $\partial R_t/\partial \tau_t^I|_{\phi_t} = -(r_t(\theta_t) - \rho)\omega_t$. With $s_t \equiv y_t + a_t - c_t$ and $\Delta_t \equiv \mathrm{E}_t[\mathcal{V}'_{t+1}(\cdot)(r_t(\theta_t) - \rho)]$, the last line is eq. (19), and the fourth line is the first equality in eq. (18). The state-contingent instruments enter through expectations that are integrals against the conditional density of the current state: $\mathrm{E}_t[\Gamma(G_t(\theta_t))] = \int_{\Theta_t} \Gamma(G_t(\theta_t)) f(\theta_t \mid \theta^{t-1})\,\mathrm{d}\theta_t$, so that perturbing public goods provision in state θ_t only changes the value function by $\Gamma'(G_t(\theta_t))f(\theta_t \mid \theta^{t-1})$ per unit, which is eq. (20); likewise, $\mathrm{E}_t[\mathcal{V}_{t+1}((y_t + a_t - c_t)R_t - T_t(\theta_t))] = \int_{\Theta_t} \mathcal{V}_{t+1}(\cdot) f(\theta_t \mid \theta^{t-1})\,\mathrm{d}\theta_t$, so that perturbing the lump-sum tax in state θ_t only changes the value function by $-(1+\beta)^{-1}\mathcal{V}'_{t+1}(\cdot)f(\theta_t \mid \theta^{t-1})$, which is eq. (21) once $\mathcal{V}'_{t+1}(a_{t+1}) = u'(c_{t+1})$ from eq. (15) at $t+1$ is used. As in the main text, $f(\theta_t)$ is shorthand for the conditional density $f(\theta_t \mid \theta^{t-1})$ against which $\mathrm{E}_t[\cdot]$ integrates; under idiosyncratic risk the two coincide.

The remaining derivatives are simplified using the first-order condition for ω_t. For an interior portfolio it reads $\mathrm{E}_t[\mathcal{V}'_{t+1}(\cdot)(r_t(\theta_t) - \rho)] = \Delta_t = 0$, i.e.

$$\frac{\mathrm{E}_t[\mathcal{V}'_{t+1}(\cdot)r_t(\theta_t)]}{\mathrm{E}_t[\mathcal{V}'_{t+1}(\cdot)]} = \rho. \tag{53}$$

Note that optimal risk-taking is not directly affected by tax parameters since taxes affect all assets symmetrically. Substituting this expression in the derivative with respect to τ_t^I yields eq.

(16):

$$\begin{aligned}\frac{\partial \mathcal{V}_t(a_t)}{\partial \tau_t^I} &= -\frac{1}{(1+\beta)}\mathrm{E}_t[\mathcal{V}'_{t+1}(\cdot)(y_t+a_t-c_t)(\rho(1-\omega_t)+r_t(\theta_t)\omega_t)] \\ &= -\frac{1}{(1+\beta)}(y_t+a_t-c_t)\mathrm{E}_t[\mathcal{V}'_{t+1}(\cdot)]\left[\rho(1-\omega_t)+\omega_t\frac{\mathrm{E}_t[\mathcal{V}'_{t+1}(\cdot)r_t(\theta_t)]}{\mathrm{E}_t[\mathcal{V}'_{t+1}(\cdot)]}\right] \\ &= -\frac{\mathrm{E}_t[\mathcal{V}'_{t+1}(\cdot)]}{(1+\beta)}(y_t+a_t-c_t)\rho,\end{aligned} \tag{54}$$

and eq. (19) equals zero. Slightly rewriting the derivative with respect to τ_t^A gives eq. (17):

$$\frac{\partial \mathcal{V}_t(a_t)}{\partial \tau_t^A} = -\frac{1}{(1+\beta)}\mathrm{E}_t[\mathcal{V}'_{t+1}(\cdot)(y_t+a_t-c_t)] = -\frac{\mathrm{E}_t[\mathcal{V}'_{t+1}(\cdot)]}{(1+\beta)}(y_t+a_t-c_t). \tag{55}$$

Finally, the second equality in eq. (18) follows from the Euler equation (7), $u'(c_t) = (1+\beta)^{-1}\mathrm{E}_t[\mathcal{V}'_{t+1}(\cdot)R_t]$, and $\mathrm{E}_t[\mathcal{V}'_{t+1}(\cdot)R_t] = \phi_t\mathrm{E}_t[\mathcal{V}'_{t+1}(\cdot)] + (1-\tau_t^I)\omega_t\Delta_t$: whenever $(1-\tau_t^I)\omega_t\Delta_t = 0$ – for an interior portfolio, or at $\tau_t^I = 1 - \mathrm{E}_t[\mathcal{V}'_{t+1}(\cdot)] = (1+\beta)u'(c_t)/\phi_t$.

Part (ii). Consider a policy instrument z_t dated t and set in history θ^{t-1}, and iterate the Bellman recursion backwards from t to 0. At any date $s < t$, the individual's choices (c_s, ω_s) are optimal, so that by the envelope theorem a change in z_t affects $\mathcal{V}_s$ only through the continuation value: $\partial\mathcal{V}_s(a_s)/\partial z_t = (1+\beta)^{-1}\mathrm{E}_s[\partial\mathcal{V}_{s+1}(a_{s+1})/\partial z_t]$, since next-period assets a_{s+1} at dates $s < t$ do not depend directly on z_t. Iterating from $s = 0$ to $s = t-1$ gives $\partial\mathcal{V}_0(a_0)/\partial z_t = (1+\beta)^{-t}\mathrm{E}_0[\partial\mathcal{V}_t(a_t)/\partial z_t]$. Since z_t is perturbed in one history θ^{t-1} only, the expectation $\mathrm{E}_0[\cdot] = \int(\cdot)f(\theta^{t-1})\,\mathrm{d}\theta^{t-1}$ reduces to the density of that history times the derivative of $\mathcal{V}_t$ in that history, which is eq. (22). Under idiosyncratic risk the policy instruments are deterministic and the same argument gives $\partial\mathcal{V}_0/\partial z_t = (1+\beta)^{-t}\partial\mathcal{V}_t/\partial z_t$, with the expectation over the individual's own history suppressed as throughout. Eqs. (23)–(26) follow by substituting eqs. (18), (19), (20) and (21) into eq. (22) and using $f(\theta^{t-1})f(\theta_t) = f(\theta^t)$; for the state-contingent instruments $G_t(\theta_t)$ and $T_t(\theta_t)$, which are set in history θ^t, the derivative of $\mathcal{V}_t$ itself carries the conditional density $f(\theta_t)$, so that the density of the history at which the instrument is set, $f(\theta^t)$, results. Public debt does not enter the individual's problem and has no direct effect on $\mathcal{V}_0$.

C Proofs comparative statics

This appendix proves the comparative statics of the compensated tax reforms in Lemmas (3), (5) and (6). These proofs exploit the fact that both reforms leave the individual's opportunity set unchanged once portfolio positions are measured in after-tax units. The proofs also demonstrate that the policy reforms keep expected utility constant, confirming the derivations via Roy's identity in the proofs of Propositions 1, 2 and 3.

A reform of the period-t tax rates only is considered. Taxes at all other dates s and the lump-sum taxes $T_s(\theta_s)$ are unchanged. The continuation value function $\mathcal{V}_{t+1}(\cdot)$ is determined by policies from period $t+1$ onwards, and is therefore unaffected by the period-t reform. Moreover, since public goods enter utility separably, individual behaviour is independent of the path of public goods provision, so that the comparative statics below do not depend on how G_t

adjusts. Throughout, we assume a nondegenerate return distribution and an interior optimum. Moreover, we only consider the cases $\tau_t^I < 1$, $\tau_t^P < 1$ and $s_t > 0$.

C.1 Idiosyncratic risk and aggregate risk with state-contingent public goods

The comparative statics for the tax reforms under idiosyncratic risk (with ex ante and ex post wealth taxes) and for aggregate risk with state-contingent lump-sum taxes are the same – up to a scaling factor with ex post wealth taxes.

First, define the after-tax gross safe return factor ϕ_t and the after-tax equity exposure $\hat{e}_t$ under the tax on capital income and ex ante wealth taxes as:

$$\phi_t \equiv 1 + (1 - \tau_t^I)\rho - \tau_t^A, \qquad \hat{e}_t \equiv (1 - \tau_t^I)\omega_t s_t. \tag{56}$$

And, similarly, define it for ex post wealth taxes as:

$$\phi_t \equiv (1 - \tau_t^P)(1 + (1 - \tau_t^I)\rho), \qquad \hat{e}_t \equiv (1 - \tau_t^P)(1 - \tau_t^I)\omega_t s_t. \tag{57}$$

In both cases, next-period assets can be written as:

$$a_{t+1} = s_t R_t - T_t(\theta_t) = \phi_t s_t + (r_t(\theta_t) - \rho)\hat{e}_t - T_t(\theta_t). \tag{58}$$

For given $s_t > 0$ and tax rates below unity, so there is a perfect mapping $(c_t, \omega_t) \mapsto (c_t, \hat{e}_t)$, so that the individual may equivalently choose $(c_t, \hat{e}_t)$ instead of (c_t, ω_t).

Second, it can be shown that the period-t tax rates enter the optimisation problem *only* through the after-tax safe return factor ϕ_t. By using $s_t = y_t + a_t - c_t$ and eq. (58), the period-t Bellman recursion in eq. (6) can be written as:

$$\mathcal{V}_t(a_t) = \max_{c_t, \hat{e}_t} u(c_t) + \mathrm{E}_t[\Gamma(G_t)] + \frac{1}{(1+\beta)}\mathrm{E}_t\left[\mathcal{V}_{t+1}\left(\phi_t(y_t + a_t - c_t) + (r_t(\theta_t) - \rho)\hat{e}_t - T_t(\theta_t)\right)\right]. \tag{59}$$

The objective is strictly concave in $(c_t, \hat{e}_t)$.[36] Hence, the optimum and the value function depend on the period-t tax rates only through ϕ_t:

$$c_t^* = c(a_t; \phi_t), \qquad \hat{e}_t^* = \hat{e}(a_t; \phi_t), \qquad \mathcal{V}_t(a_t) = \mathcal{V}(a_t; \phi_t). \tag{60}$$

Third, all tax reforms hold the after tax safe return ϕ_t constant. Under the reform with ex ante wealth taxes in Lemma (1): $\mathrm{d}\phi_t = -\rho \mathrm{d}\tau_t^I - \mathrm{d}\tau_t^A = 0$. Under the reform with ex post wealth taxes in Lemma (4): $\mathrm{d}\phi_t = -(1 + (1 - \tau_t^I)\rho)\mathrm{d}\tau_t^P - (1 - \tau_t^P)\rho \mathrm{d}\tau_t^I = 0$. It follows immediately that under either reform: (i) optimal consumption is unchanged, and hence saving is unchanged, $\mathrm{d}s_t = \mathrm{d}(y_t + a_t - c_t) = 0$, so that $\varepsilon_{s\tau^I} = 0$; (ii) the after-tax equity exposure is unchanged, $\mathrm{d}\hat{e}_t = 0$ – the individual attains the same combination of net returns and risk as before the reform; (iii) the value function is unchanged, $\mathrm{d}\mathcal{V}_t(a_t) = 0$ – the reform is compensated and

[36] The argument of $\mathcal{V}_{t+1}$ is affine in $(c_t, \hat{e}_t)$, $u(\cdot)$ is strictly concave, and $\mathcal{V}_{t+1}(\cdot)$ is strictly concave. Strict concavity of $\mathcal{V}_{t+1}(\cdot)$ follows from strict concavity of $u(\cdot)$ by standard arguments, see also Alvarez and Stokey (1998). Strict concavity of the maximand in eq. (59) in $(c_t, \hat{e}_t)$ then follows. Hence, optimal choices for c_t and $\hat{e}_t$ are unique.

keeps expected utility constant; and (iv) by eq. (58), next-period assets a_{t+1} are unchanged in every state θ_t, so that the reform has no effects in periods after t, while the invariance of $\mathcal{V}_t(\cdot)$ as a function implies that the reform has no effects in periods before t either.

Fourth, and final, the policy elasticities can be derived. Under the ex ante reform, $\mathrm{d}[(1-\tau_t^I)\omega_t s_t]=0$ and $\mathrm{d}s_t=0$ imply $(1-\tau_t^I)\mathrm{d}\omega_t=\omega_t\mathrm{d}\tau_t^I$, so that:

$$\varepsilon_{\omega\tau^I}\equiv\frac{\mathrm{d}\omega_t}{\mathrm{d}\tau_t^I}\frac{(1-\tau_t^I)}{\omega_t}=1, \tag{61}$$

which is the Domar-Musgrave result in Lemma (3). Under the ex post reform, $\mathrm{d}[(1-\tau_t^P)(1-\tau_t^I)\omega_t s_t]=0$ and $\mathrm{d}s_t=0$ imply:

$$\frac{\mathrm{d}\omega_t}{\omega_t}=\frac{\mathrm{d}\tau_t^I}{1-\tau_t^I}+\frac{\mathrm{d}\tau_t^P}{1-\tau_t^P}=\frac{\mathrm{d}\tau_t^I}{1-\tau_t^I}-\frac{\rho\mathrm{d}\tau_t^I}{1+(1-\tau_t^I)\rho}=\frac{\mathrm{d}\tau_t^I}{(1-\tau_t^I)(1+(1-\tau_t^I)\rho)}, \tag{62}$$

where the second step substitutes the ex post reform of Lemma (4). Hence:

$$\varepsilon_{\omega\tau^I}=\frac{1}{1+(1-\tau_t^I)\rho}<1. \tag{63}$$

This establishes Lemma (5).

C.2 Aggregate risk with state-contingent lump-sum taxes

In the case of aggregate risk wih state-contingent lump-sum taxes, the behavioural responses are simple. First, totally differentiate the return factor $R_t=1+(1-\tau_t^I)\rho-\tau_t^A+(1-\tau_t^I)(r_t(\theta_t)-\rho)\omega_t$ and impose the tax reform ($\mathrm{d}\tau_t^A=-\rho\mathrm{d}\tau_t^I$) to find:

$$\mathrm{d}R_t=(r_t(\theta_t)-\rho)\left[(1-\tau_t^I)\mathrm{d}\omega_t-\omega_t\mathrm{d}\tau_t^I\right]. \tag{64}$$

Second, totally differentiate the asset accumulation equation $a_{t+1}=R_t s_t-T_t(\theta_t)$ and substitute eqs. (64) and (83) to obtain:

$$\mathrm{d}a_{t+1}=(1+\rho+(r_t(\theta_t)-\rho)\omega_t)\,\mathrm{d}s_t+(r_t(\theta_t)-\rho)s_t\mathrm{d}\omega_t, \tag{65}$$

while totally differentiating the period budget constraint $c_t=y_t+a_t-s_t$, with a_t predetermined and y_t exogenous, gives $\mathrm{d}c_t=-\mathrm{d}s_t$. Importantly, every tax rate – and the tax reform itself – has dropped out of the perturbed dynamic budget constraint (65). Intuitively, the individual earns the *pre-tax* return $1+\rho+(r_t(\theta_t)-\rho)\omega_t$ in every state. The tax reform merely changes which part of this return is received directly and which part is received indirectly via the state contingent transfer $T_t(\theta_t)$.

Third, optimal portfolio choice and savings do not change. To prove this, set $\mathrm{d}s_t=\mathrm{d}\omega_t=0$. Then eq. (65) gives $\mathrm{d}a_{t+1}=0$ in every state θ_t, so that no effects spill over to period $t+1$ or beyond. Hence, the value function $\mathcal{V}_{t+1}(\cdot)$ is unchanged as of date t. The Euler equation (7) is unaffected because the after-tax safe return ϕ_t is unchanged by construction of the reform. The portfolio condition (8) is unaffected because does not depend on $\tau_t^I>0$ if a_{t+1} remains constant. Consequently, there are no behavioural changes at all: $\mathrm{d}c_t=\mathrm{d}s_t=\mathrm{d}\omega_t=\mathrm{d}a_{t+1}=0$.

In contrast to Propositions 1 and 2 the Domar-Musgrave effect vanishes, because the reduction in the household's private risk exposure due to the tax reform is matched by an increase in risk due to the state-contingent rebate.

D Proof Proposition 1

Total utility remains constant under the reform, which can be established by totally differentiating the Bellman recursion at time t in equation (6):

$$\mathrm{d}\mathcal{V}_t(a_t) = \frac{\partial \mathcal{V}_t(a_t)}{\partial \tau_t^I}\mathrm{d}\tau_t^I + \frac{\partial \mathcal{V}_t(a_t)}{\partial \tau_t^A}\mathrm{d}\tau_t^A. \tag{66}$$

After substituting the derivatives of the value function from Lemma 2 we find:

$$\mathrm{d}\mathcal{V}_t(a_t) = \left[-\frac{\mathrm{E}_t[\mathcal{V}'_{t+1}(.)]}{(1+\beta)}(y_t + a_t - c_t)\right](\rho \mathrm{d}\tau_t^I + \mathrm{d}\tau_t^A) = 0, \tag{67}$$

where we substituted the tax reform that keeps the intertemporal price of consumption fixed ($\mathrm{d}\tau_t^A = -\rho \mathrm{d}\tau_t^I$) in the last step. Hence, the value function does not change under the tax reform and expected utility remains constant.

Totally differentiating the government budget constraint – while keeping public spending constant – then gives:

$$\mathrm{d}\mathcal{R}_{t+1} = (\rho + \pi_t\omega_t)s_t\mathrm{d}\tau_t^I + s_t\mathrm{d}\tau_t^A + (\tau_t^I(\rho + \pi_t\omega_t) + \tau_t^A)\mathrm{d}s_t + \tau_t^I s_t \pi_t \mathrm{d}\omega_t. \tag{68}$$

Given that we have a partial-equilibrium set-up, the risk-premium remains unaffected by a marginal change in tax policy ($\mathrm{d}\pi_t = 0$). Substituting for the tax reform (i.e. $\mathrm{d}\tau_t^A = -\rho \mathrm{d}\tau_t^I$) and rewriting yields:

$$\frac{\mathrm{d}\mathcal{R}_{t+1}/\mathrm{d}\tau_t^I}{s_t} = \pi_t\omega_t - \frac{(\tau_t^I(\rho + \pi_t\omega_t) + \tau_t^A)}{1-\tau_t^I}\varepsilon_{s\tau^I} + \frac{\tau_t^I}{1-\tau_t^I}\pi_t\omega_t\varepsilon_{\omega\tau^I}. \tag{69}$$

Finally, imposing the comparative statics of Lemma (3) ($\varepsilon_{s\tau^I} = 0$, $\varepsilon_{\omega\tau^I} = 1$) delivers the expression in Proposition 1. This completes the proof.

D.1 Rebate the revenue via a change in public goods

Footnote 28 claims that it does not matter whether deterministic revenue is rebated via transfers or public goods. This appendix demonstrates this formally. Additional revenue from the reform is collected in period $t+1$ and can be rebated through larger public goods provision in period $t+1$. Since G_{t+1} enters the individual's problem through $\mathcal{V}_{t+1}$, totally differentiating the Bellman recursion at time t in equation (6) gives:

$$\mathrm{d}\mathcal{V}_t(a_t) = \frac{\partial \mathcal{V}_t(a_t)}{\partial \tau_t^I}\mathrm{d}\tau_t^I + \frac{\partial \mathcal{V}_t(a_t)}{\partial \tau_t^A}\mathrm{d}\tau_t^A + \frac{1}{(1+\beta)}\Gamma'(G_{t+1})\mathrm{d}G_{t+1}. \tag{70}$$

Substituting the derivatives of the value function from Lemma 2 and the tax reform ($\mathrm{d}\tau_t^A =$

$-\rho \mathrm{d}\tau_t^I$) yields:

$$\mathrm{d}\mathcal{V}_t(a_t) = \frac{\Gamma'(G_{t+1})}{1+\beta}\mathrm{d}G_{t+1}. \tag{71}$$

Totally differentiating the government budget constraint, while allowing period-$t+1$ public spending to change, then gives (note that $\mathrm{d}\mathcal{R}_{t+1} = 0$):

$$\mathrm{d}G_{t+1} = (\rho + \pi_t\omega_t)s_t\mathrm{d}\tau_t^I + s_t\mathrm{d}\tau_t^A + (\tau_t^I(\rho + \pi_t\omega_t) + \tau_t^A)\mathrm{d}s_t + \tau_t^I s_t \pi_t \mathrm{d}\omega_t. \tag{72}$$

Substituting for the tax reform and rewriting yields:

$$\frac{\mathrm{d}G_{t+1}/\mathrm{d}\tau_t^I}{s_t} = \pi_t\omega_t - \frac{(\tau_t^I(\rho + \pi_t\omega_t) + \tau_t^A)}{1-\tau_t^I}\varepsilon_{s\tau^I} + \frac{\tau_t^I}{1-\tau_t^I}\pi_t\omega_t\varepsilon_{\omega\tau^I}. \tag{73}$$

This implies that the conclusion is independent from whether the rebate of larger tax revenue is via a lump-sum transfer or via larger public goods provision.

E Proof Proposition 2

Total utility remains constant under the reform, which can be established by totally differentiating the Bellman recursion at time t in equation (6):

$$\mathrm{d}\mathcal{V}_t(a_t) = \frac{\partial \mathcal{V}_t(a_t)}{\partial \tau_t^I}\mathrm{d}\tau_t^I + \frac{\partial \mathcal{V}_t(a_t)}{\partial \tau_t^P}\mathrm{d}\tau_t^P. \tag{74}$$

Substituting the derivatives of the value function from the equivalent of Lemma 2 for ex post wealth taxes yields:

$$\mathrm{d}\mathcal{V}_t(a_t) = \left[-\frac{\mathrm{E}_t[\mathcal{V}'_{t+1}(.)]}{(1+\beta)}(y_t + a_t - c_t)\right]\left(\frac{(1-\tau_t^P)\rho}{1+(1-\tau_t^I)\rho}\mathrm{d}\tau_t^I + \mathrm{d}\tau_t^P\right) = 0, \tag{75}$$

where the tax reform is substituted that keeps the intertemporal price of consumption fixed ($\mathrm{d}\tau_t^P = -\frac{(1-\tau_t^P)\rho}{1+(1-\tau_t^I)\rho}\mathrm{d}\tau_t^I$) in the last step. Hence, the value function does not change under the tax reform and expected utility remains constant.

Totally differentiating the government budget constraint – while keeping public spending constant – then gives:

$$\begin{aligned}\mathrm{d}\mathcal{R}_{t+1} \;=\; & (1-\tau_t^P)(\rho + \pi_t\omega_t)s_t\mathrm{d}\tau_t^I + (1 + (1-\tau_t^I)(\rho + \pi_t\omega_t))s_t\mathrm{d}\tau_t^P \\ & + (\tau_t^I(\rho + \pi_t\omega_t) + \tau_t^P(1 + (1-\tau_t^I)(\rho + \pi_t\omega_t)))\mathrm{d}s_t \\ & + (\tau_t^I\pi_t s_t + \tau_t^P(1-\tau_t^I)\pi_t s_t)\mathrm{d}\omega_t\end{aligned} \tag{76}$$

Like before, the risk-premium remains unaffected by a marginal change in tax policy ($\mathrm{d}\pi_t = 0$).

Substituting for the tax reform (i.e. $\mathrm{d}\tau_t^P = -\frac{(1-\tau_t^P)\rho}{1+(1-\tau_t^I)\rho}\mathrm{d}\tau_t^I$) and rewriting yields:

$$\begin{aligned}\frac{\mathrm{d}\mathcal{R}_{t+1}/\mathrm{d}\tau_t^I}{s_t} &= \frac{(1-\tau_t^P)\pi_t\omega_t}{1+(1-\tau_t^I)\rho} - \frac{(\tau_t^I(\rho+\pi_t\omega_t)+\tau_t^P(1+(1-\tau_t^I)(\rho+\pi_t\omega_t)))}{(1-\tau_t^I)}\varepsilon_{s\tau^I} \\ &\quad + \frac{(\tau_t^I+\tau_t^P(1-\tau_t^I))}{(1-\tau_t^I)}\pi_t\omega_t\varepsilon_{\omega\tau^I}.\end{aligned} \tag{77}$$

Substituting $\varepsilon_{s\tau^I} = 0$ and $\varepsilon_{\omega\tau^I} = 1/(1+(1-\tau_t^I)\rho)$ from Lemma (5) and using $(1-\tau_t^P)(1-\tau_t^I)+\tau_t^I+\tau_t^P(1-\tau_t^I) = 1$ proves the proposition.

F Proof Proposition 3

This appendix establishes Proposition 3 (neutrality of the reform under aggregate risk with state-contingent lump-sum taxes). Under the timing conventions of the model, the stochastic revenue from the period-t tax bases is collected at the beginning of period $t+1$ and is absorbed by state-contingent lump-sum taxes in period $t+1$: $\mathrm{d}T_t(\theta_t)$ depends on the realisation of θ_t, $\mathrm{d}\mathcal{R}_{t+1} = 0$, and the public goods provision tax is held constant ($\mathrm{d}G_{t+1} = 0$ for all θ_t).

Totally differentiating the period-t value function gives:

$$\mathrm{d}\mathcal{V}_t(a_t) = \frac{\partial\mathcal{V}_t(a_t)}{\partial\tau_t^I}\mathrm{d}\tau_t^I + \frac{\partial\mathcal{V}_t(a_t)}{\partial\tau_t^A}\mathrm{d}\tau_t^A + \int_{\Theta_t}\frac{\partial\mathcal{V}_t(a_t)}{\partial T_t(\theta_t)}\mathrm{d}T_t(\theta_t)\mathrm{d}\theta_t. \tag{78}$$

Note that the lump-sum tax is different in each state θ_t, since the government budget is stochastic, so that the last term sums the effects of the state-by-state changes $\mathrm{d}T_t(\theta_t)$ over all states. Substituting the derivatives of the value function from Lemma (2) – where eq. (21) carries the density $f(\theta_t)$, so that the integral over states becomes a conditional expectation – gives:

$$\mathrm{d}\mathcal{V}_t(a_t) = \left[-\frac{\mathrm{E}_t[\mathcal{V}'_{t+1}(.)]}{(1+\beta)}(y_t+a_t-c_t)\right](\rho\mathrm{d}\tau_t^I+\mathrm{d}\tau_t^A) - \frac{1}{(1+\beta)}\mathrm{E}_t[\mathcal{V}'_{t+1}(.)\mathrm{d}T_t(\theta_t)]. \tag{79}$$

Substituting the tax reform ($\mathrm{d}\tau_t^A = -\rho\mathrm{d}\tau_t^I$) yields that the first term vanishes (as in the proofs of Propositions 1 and 2):

$$\mathrm{d}\mathcal{V}_t(a_t) = -\frac{1}{(1+\beta)}\mathrm{E}_t[\mathcal{V}'_{t+1}(.)\mathrm{d}T_t(\theta_t)]. \tag{80}$$

Totally differentiating the government budget constraint, while allowing the lump-sum tax to adjust, keeping public goods constant, and imposing budget balance, gives for each realisation θ_t:

$$\mathcal{R}_{t+1} = (1+\rho)d_{t+1} - d_{t+2} = \tau_t^I(\rho+(r_t(\theta_t)-\rho)\omega_t)s_t + \tau_t^A s_t + T_t(\theta_t) - G_{t+1}. \tag{81}$$

$$\begin{aligned}0 &= \mathrm{d}\mathcal{R}_{t+1} = (\rho+(r_t(\theta_t)-\rho)\omega_t)s_t\mathrm{d}\tau_t^I + s_t\mathrm{d}\tau_t^A \\ &+ (\tau_t^I(\rho+(r_t(\theta_t)-\rho)\omega_t)+\tau_t^A)\mathrm{d}s_t + \tau_t^I s_t(r_t(\theta_t)-\rho)\mathrm{d}\omega_t + \mathrm{d}T_t(\theta_t).\end{aligned} \tag{82}$$

Imposing the tax reform ($\mathrm{d}\tau_t^A = -\rho\mathrm{d}\tau_t^I$) and solving for the required change in the lump-sum

tax in each state θ_t gives:

$$\mathrm{d}T_t(\theta_t) = -(r_t(\theta_t)-\rho)\omega_t s_t \mathrm{d}\tau_t^I - (\tau_t^I(\rho + (r_t(\theta_t)-\rho)\omega_t) + \tau_t^A)\mathrm{d}s_t - \tau_t^I s_t (r_t(\theta_t)-\rho)\mathrm{d}\omega_t. \quad (83)$$

Substituting eq. (83) into eq. (80), noting that $\mathrm{d}s_t$ and $\mathrm{d}\omega_t$ are known at t and can be taken outside the expectation, and collecting all terms that are proportional to the excess return $r_t(\theta_t) - \rho$, yields:

$$\begin{aligned} \mathrm{d}\mathcal{V}_t(a_t) &= \frac{\left(\omega_t s_t \mathrm{d}\tau_t^I + \tau_t^I \omega_t \mathrm{d}s_t + \tau_t^I s_t \mathrm{d}\omega_t\right)}{(1+\beta)} \mathrm{E}_t[\mathcal{V}'_{t+1}(.)(r_t(\theta_t)-\rho)] \quad (84) \\ &\quad + \frac{(\tau_t^I \rho + \tau_t^A)\mathrm{d}s_t}{(1+\beta)} \mathrm{E}_t[\mathcal{V}'_{t+1}(.)]. \end{aligned}$$

The first term is zero – *irrespective of the portfolio response* $\mathrm{d}\omega_t$ and the saving response $\mathrm{d}s_t$ – by the first-order condition for optimal portfolio choice in eq. (8), since it implies $\mathrm{E}_t[\mathcal{V}'_{t+1}(.)(r_t(\theta_t)-\rho)] = 0$. Hence:

$$\mathrm{d}\mathcal{V}_t(a_t) = \frac{\mathrm{E}_t[\mathcal{V}'_{t+1}(.)]}{(1+\beta)}(\tau_t^I \rho + \tau_t^A)\mathrm{d}s_t. \quad (85)$$

From Appendix C.2 follows that $\mathrm{d}s_t = 0$, so that eq. (85) gives:

$$\frac{\mathrm{d}\mathcal{V}_t(a_t)/\mathrm{d}\tau_t^I}{u'(c_t)s_t} = 0. \quad (86)$$

Moreover, from Appendix C.2 also follows that $\mathrm{d}\omega_t = 0$, so no portfolio reshuffling occurs. This completes the proof.

G Proof Proposition 4

This appendix establishes Proposition 4 (the welfare effect of the reform under aggregate risk with state-contingent public goods provision). The steps parallel the proof of Proposition 3 in the previous appendix, with the state-contingent public goods G_{t+1} taking over the role of the state-contingent lump-sum tax $T_t(\theta_t)$, which is held constant ($\mathrm{d}T_t = 0$ for all θ_t).

Under the symmetric timing, the stochastic revenue from the period-t tax bases is collected at the beginning of period $t+1$ and is absorbed by state-contingent public goods provision in period $t+1$: $\mathrm{d}G_{t+1}$ depends on the realisation of θ_t, $\mathrm{d}\mathcal{R}_{t+1} = 0$, and the lump-sum tax is held constant ($\mathrm{d}T_t(\theta_t) = 0$ for all θ_t).

Totally differentiating the period-t value function, substituting the tax reform ($\mathrm{d}\tau_t^A = -\rho \mathrm{d}\tau_t^I$) (as in the proof of Proposition 1), and using that G_{t+1} enters $\mathcal{V}_t$ through $\mathcal{V}_{t+1}$, yields:

$$\mathrm{d}\mathcal{V}_t(a_t) = \frac{1}{(1+\beta)} \mathrm{E}_t[\Gamma'(G_{t+1})\mathrm{d}G_{t+1}]. \quad (87)$$

Totally differentiating the government budget constraint (12), while allowing period-$t+1$

public goods to adjust and imposing budget balance, gives for each realisation θ_t:

$$\begin{aligned} \mathrm{d}G_{t+1} &= (\rho + (r_t(\theta_t) - \rho)\omega_t) s_t \mathrm{d}\tau_t^I + s_t \mathrm{d}\tau_t^A \\ &\quad + (\tau_t^I(\rho + (r_t(\theta_t) - \rho)\omega_t) + \tau_t^A)\mathrm{d}s_t + \tau_t^I s_t (r_t(\theta_t) - \rho)\mathrm{d}\omega_t. \end{aligned} \tag{88}$$

Imposing the tax reform ($\mathrm{d}\tau_t^A = -\rho \mathrm{d}\tau_t^I$) and the comparative statics of Lemma (3) ($\mathrm{d}s_t = 0$ and $\mathrm{d}\omega_t / \mathrm{d}\tau_t^I = \omega_t/(1-\tau_t^I)$) gives the response of public goods provision in each state θ_t:

$$\frac{\mathrm{d}G_{t+1}}{\mathrm{d}\tau_t^I} = (r_t(\theta_t) - \rho)\omega_t s_t + \tau_t^I s_t (r_t(\theta_t) - \rho)\frac{\omega_t}{1-\tau_t^I} = \frac{(r_t(\theta_t) - \rho)\omega_t s_t}{1-\tau_t^I}. \tag{89}$$

Public goods provision rises in states where the realised return exceeds the safe return ($r_t(\theta_t) > \rho$) and falls in states where it falls short of it, since the revenue from taxing the risk premium is itself stochastic. Taking expectations of eq. (88), imposing the tax reform, and using $\mathrm{d}\pi_t = 0$ in our partial-equilibrium setting, gives the expected revenue effect – using the same steps as in the proof of Proposition 1:

$$\frac{\mathrm{E}_t[\mathrm{d}G_{t+1}]/\mathrm{d}\tau_t^I}{s_t} = \pi_t \omega_t - \frac{(\tau_t^I(\rho + \pi_t\omega_t) + \tau_t^A)}{1-\tau_t^I}\varepsilon_{s\tau^I} + \frac{\tau_t^I}{1-\tau_t^I}\pi_t\omega_t\varepsilon_{\omega\tau^I}. \tag{90}$$

Add and subtract $\mathrm{E}_t[u'(c_{t+1})]$ in eq. (87):

$$\mathrm{d}\mathcal{V}_t(a_t) = \frac{1}{(1+\beta)}\mathrm{E}_t\left[\left(\Gamma'(G_{t+1}) - \mathrm{E}_t[u'(c_{t+1})]\right)\mathrm{d}G_{t+1}\right] + \frac{\mathrm{E}_t[u'(c_{t+1})]}{(1+\beta)}\mathrm{E}_t[\mathrm{d}G_{t+1}]. \tag{91}$$

The first term of eq. (91) values the state-by-state response of public goods provision at the difference between the marginal utility of public goods and the expected marginal utility of private consumption; the second term values the expected response at the expected marginal utility of private consumption.

Substituting eqs. (89) and (90) into eq. (91), dividing by $u'(c_t)s_t$, and using $(1+\beta)u'(c_t) = \phi_t \mathrm{E}_t[u'(c_{t+1})]$ – which follows from the Euler equation (7) and the portfolio condition (8) with $\mathcal{V}'_{t+1}(\cdot) = u'(c_{t+1})$ – yields the first two lines of eq. (35) in Proposition 4:

$$\begin{aligned} \frac{\phi_t \mathrm{d}\mathcal{V}_t(a_t)/\mathrm{d}\tau_t^I}{u'(c_t)s_t} &= \pi_t\omega_t - \frac{(\tau_t^I(\rho + \pi_t\omega_t) + \tau_t^A)}{1-\tau_t^I}\varepsilon_{s\tau^I} + \frac{\tau_t^I}{1-\tau_t^I}\pi_t\omega_t\varepsilon_{\omega\tau^I} \\ &\quad + \frac{\omega_t}{1-\tau_t^I}\mathrm{E}_t\left[\left(\frac{\Gamma'(G_{t+1})}{\mathrm{E}_t[u'(c_{t+1})]} - 1\right)(r_t(\theta_t) - \rho)\right]. \end{aligned} \tag{92}$$

Next, the last term of eq. (92) is rewritten. Since $\mathrm{E}_t[u'(c_{t+1})]$ is measurable with respect to the period-t information set, it is a constant under the outer expectation $\mathrm{E}_t[\cdot]$ and factors out. Using the decomposition of an expected product in a product of expectations and a covariance, i.e. $\mathrm{E}_t[XY] = \mathrm{E}_t[X]\mathrm{E}_t[Y] + \mathrm{cov}_t[X,Y]$, with $X = \Gamma'(G_{t+1})/\mathrm{E}_t[u'(c_{t+1})] - 1$ and $Y = r_t(\theta_t) - \rho$, and using $\mathrm{E}_t[r_t(\theta_t) - \rho] = \pi_t$ as well as $\mathrm{cov}_t[X,Y] = \mathrm{cov}_t[\Gamma'(G_{t+1}), r_t(\theta_t)]/\mathrm{E}_t[u'(c_{t+1})]$, gives:

$$\mathrm{E}_t\left[\left(\frac{\Gamma'(G_{t+1})}{\mathrm{E}_t[u'(c_{t+1})]} - 1\right)(r_t(\theta_t) - \rho)\right] = \pi_t\left(\frac{\mathrm{E}_t[\Gamma'(G_{t+1})]}{\mathrm{E}_t[u'(c_{t+1})]} - 1\right) - \pi_t^G\frac{\mathrm{E}_t[\Gamma'(G_{t+1})]}{\mathrm{E}_t[u'(c_{t+1})]}, \tag{93}$$

where $\pi_t^G \equiv -\frac{\text{cov}_t[r_t(\theta_t),\Gamma'(G_{t+1})]}{\text{E}_t[\Gamma'(G_{t+1})]}$ is the public risk premium. Substituting eq. (93) in eq. (92) and imposing the comparative statics of Lemma (3) ($\varepsilon_{s\tau^I} = 0$, $\varepsilon_{\omega\tau^I} = 1$) delivers the last line of eq. (35) in Proposition 4. This completes the proof.

H Proof Proposition 5

This appendix derives the optimal taxes on capital income and wealth under idiosyncratic risk. The government's instruments are $\{\tau_t^I, \phi_t, G_{t+1}, d_{t+2}\}$ for all $t \geq 0$, with $\phi_t \equiv 1+(1-\tau_t^I)\rho-\tau_t^A$. Saving s_t, the portfolio share ω_t and the gross equity exposure $x_t \equiv \omega_t s_t$ are functions of the entire policy path. Under idiosyncratic risk all aggregates are deterministic by the exact law of large numbers. Hence, the expectation E_0 over individual histories is suppressed throughout. Every sign argument below holds history by history and, at the optimum, full insurance makes all individuals identical.

For reasons that become clear below, the no-leverage constraint $\omega_t \leq 1$ will be a binding constraint on individual's portfolio choices under the optimal tax system. The constraint is also what makes the government's problem well-defined. Without it, the Domar-Musgrave effect implies $\omega_t = \omega_t^0(1-\tau_t^{I,0})/(1-\tau_t^I)$, so that the equity tax base $x_t = \omega_t s_t$ and the revenue $\tau_t^I \pi_t x_t$ from taxing the risk premium grows without bound as τ_t^I approaches one, while the compensated reform leaves the individual's expected utility unchanged. Social welfare would then be unbounded above and no optimum would exist. Maximising the Bellman equation over ω_t subject to $\omega_t \leq 1$ (with associated Kuhn-Tucker multiplier ξ_t) gives the individual's Kuhn-Tucker conditions:

$$(1-\tau_t^I)s_t\Delta_t = (1+\beta)\xi_t, \quad \xi_t \geq 0, \quad 1-\omega_t \geq 0, \quad \xi_t(1-\omega_t) = 0, \quad \Delta_t \equiv \text{E}_t[\mathcal{V}'_{t+1}(\cdot)(r_t(\theta_t)-\rho)]. \tag{94}$$

If the equity share is interior ($\omega_t < 1$), then $\Delta_t = 0$. This is the portfolio condition (8) of the main text. If the no-leverage constraint binds ($\omega_t = 1$), then $\Delta_t \geq 0$: the individual would like to hold more equity, but cannot.

Two properties of Δ_t are used below. First, since $\mathcal{V}'_{t+1}$ is decreasing in a_{t+1} and a_{t+1} is increasing in r_t (for $\tau_t^I < 1$), $\text{cov}_t[\mathcal{V}'_{t+1}, r_t] \leq 0$, so that

$$\Delta_t = \pi_t \text{E}_t[\mathcal{V}'_{t+1}] + \text{cov}_t[\mathcal{V}'_{t+1}, r_t] \leq \pi_t \text{E}_t[\mathcal{V}'_{t+1}]. \tag{95}$$

Second, at $\tau_t^I = 1$ the individual is fully insured (which is part of the tax optimum). Then, $\mathcal{V}'_{t+1}$ is deterministic and equal to $u'(c_{t+1})$, so that eq. (95) holds with equality: $\Delta_t = \pi_t u'(c_{t+1}) > 0$. Since $\Delta_t > 0$ for τ_t^I close to 1, the individual then strictly prefers the corner of the equity share, $\omega_t = 1$. At $\tau_t^I = 1$, the individual is indifferent over ω_t, so $\omega_t = 1$ is set by continuity. The no-leverage corner features in Roy's identity for τ_t^I (through $\Delta_t \geq 0$ instead of $\Delta_t = 0$) and in the comparative statics (the Domar-Musgrave response is now switched off).

The optimal tax system is solved by maximising social welfare – utility of the representative individual – subject to the government budget constraint and an inequality constraint that taxes on capital income cannot exceed 100 percent. Attach deterministic Lagrange multiplier $(1+\beta)^{-(t+1)}\mu_{t+1}$ to the period-$t+1$ budget constraint and deterministic Lagrange multiplier

$(1+\beta)^{-(t+1)}\kappa_{t+1}$ to the upper bound on the capital income tax $\tau_t^I \leq 1$. The Lagrangian for maximising social welfare is then given by:

$$\mathcal{L} = \mathcal{V}_0(a_0) + \sum_{t=0}^{\infty} \frac{1}{(1+\beta)^{t+1}} \Big[\mu_{t+1} \Big((1+\rho-\phi_t) s_t + \tau_t^I \pi_t x_t + T_t + d_{t+2} - (1+\rho) d_{t+1} - G_{t+1} \Big) + \kappa_{t+1} \left(1 - \tau_t^I\right) \Big], \tag{96}$$

The necessary first-order conditions for optimal taxes are found by differentiating eq. (96) with respect to all policy instruments:

$$\frac{\partial \mathcal{L}}{\partial \tau_t^I} = 0: \quad \left.\frac{\partial \mathcal{V}_0}{\partial \tau_t^I}\right|_{\phi_t} + \frac{\mu_{t+1}\pi_t x_t - \kappa_{t+1}}{(1+\beta)^{t+1}} + \sum_{s=0}^{\infty} \frac{\mu_{s+1}}{(1+\beta)^{s+1}} \left[(1+\rho-\phi_s) \left.\frac{\partial s_s}{\partial \tau_t^I}\right|_{\phi_t} + \tau_s^I \pi_s \left.\frac{\partial x_s}{\partial \tau_t^I}\right|_{\phi_t} \right] = 0, \tag{97}$$

$$\frac{\partial \mathcal{L}}{\partial \phi_t} = 0: \quad \frac{\partial \mathcal{V}_0}{\partial \phi_t} - \frac{\mu_{t+1} s_t}{(1+\beta)^{t+1}} + \sum_{s=0}^{\infty} \frac{\mu_{s+1}}{(1+\beta)^{s+1}} \left[(1+\rho-\phi_s) \frac{\partial s_s}{\partial \phi_t} + \tau_s^I \pi_s \frac{\partial x_s}{\partial \phi_t} \right] = 0, \tag{98}$$

$$\frac{\partial \mathcal{L}}{\partial G_{t+1}} = 0: \quad \frac{\partial \mathcal{V}_0}{\partial G_{t+1}} - \frac{\mu_{t+1}}{(1+\beta)^{t+1}} = 0, \tag{99}$$

$$\frac{\partial \mathcal{L}}{\partial d_{t+1}} = 0: \quad \frac{\mu_t}{(1+\beta)^t} - \frac{(1+\rho)\mu_{t+1}}{(1+\beta)^{t+1}} = 0, \tag{100}$$

$$\kappa_{t+1} \geq 0, \qquad 1 - \tau_t^I \geq 0, \qquad \kappa_{t+1}(1-\tau_t^I) = 0, \tag{101}$$

where the following properties have been used: (i) since public goods are additively separable and lump-sum taxes are fixed, s_s and x_s do not depend on G or d; (ii) since (τ_t^I, ϕ_t) is a bijective reparametrisation of (τ_t^I, τ_t^A), the first-order conditions with respect to τ_t^I at a fixed ϕ_t (i.e. jointly with $\mathrm{d}\tau_t^A = -\rho \mathrm{d}\tau_t^I$) and with respect to ϕ_t together characterise the optimum over (τ_t^I, τ_t^A); hence, in eq. (97) all derivatives with respect to τ_t^I are taken at a fixed ϕ_t. (iii) In eq. (100), d_{t+1} appears with coefficient $+1$ in the period-t constraint (weight $(1+\beta)^{-t}\mu_t$) and with coefficient $-(1+\rho)$ in the period-$t+1$ constraint (weight $(1+\beta)^{-(t+1)}\mu_{t+1}$).

To simplify the first-order conditions, Lemma (2) is applied. Under idiosyncratic risk the policy instruments are deterministic, so the dynamic Roy identity (22) reads $\partial \mathcal{V}_0/\partial z_t = (1+\beta)^{-t} \partial \mathcal{V}_t/\partial z_t$, and the relevant period-t partial derivatives are eqs. (18), (19) and – for the deterministic public goods G_{t+1}, which enter $\mathcal{V}_t$ through $\mathcal{V}_{t+1}$ – $\partial \mathcal{V}_t/\partial G_{t+1} = \Gamma'(G_{t+1})/(1+\beta)$ by eq. (20) applied at $t+1$; equivalently, $\partial \mathcal{V}_0/\partial G_{t+1} = (1+\beta)^{-(t+1)}\Gamma'(G_{t+1})$ by eq. (25). Public debt has no direct effect on $\mathcal{V}_t$. Note that eq. (19) equals zero *only* when the portfolio share is interior ($\Delta_t = 0$). With a binding no-leverage constraint, it is equal to $-x_t\Delta_t/(1+\beta) \leq 0$. Hence, the compensated tax increase then has a first-order utility cost, because the individual can no longer change its risk exposure through portfolio adjustments.

Further, under the particular policy parameterisation, the comparative statics are as follows. If the no-leverage constraint is slack ($\omega_t < 1$), the reform leaves the individual's opportunity set unchanged (the after-tax safe return is fixed, and any after-tax exposure remains attainable

through ω_t), so that the entire allocation is unchanged and only the portfolio share adjusts:

$$\left.\frac{\partial s_s}{\partial \tau_t^I}\right|_{\phi_t} = 0 \ \forall s, \quad \left.\frac{\partial x_t}{\partial \tau_t^I}\right|_{\phi_t} = \frac{x_t}{1-\tau_t^I}, \quad \left.\frac{\partial x_s}{\partial \tau_t^I}\right|_{\phi_t} = 0, \ \forall s \neq t. \tag{102}$$

This is just a replication of Lemma (3) in the main text. However, if the no-leverage constraint binds, $x_t = s_t$ and ω_t can no longer rise in response to a larger τ_t^I. As a result, the after-tax exposure $(1-\tau_t^I)s_t$ falls, the opportunity set changes, and saving responds in period t and in all other periods $s \neq t$:

$$\left.\frac{\partial x_t}{\partial \tau_t^I}\right|_{\phi_t} = \left.\frac{\partial s_t}{\partial \tau_t^I}\right|_{\phi_t}, \quad \left.\frac{\partial s_s}{\partial \tau_t^I}\right|_{\phi_t} \gtrless 0. \tag{103}$$

The responses to ϕ_t, $\partial s_s/\partial \phi_t$ and $\partial x_s/\partial \phi_t$, are uncompensated and generally nonzero in all periods. Below it is shown that the characterisation of the optimum does not require a full-fledged derivation of these behavioural responses. Moreover, the comparative statics are highly complex, since saving and portfolio behaviour depend on the entire time sequence of taxes on capital income and wealth.

The first-order conditions can be simplified by substituting eqs. (18) and (19) and the dynamic Roy identity (22) into eqs. (97)–(100) and multiplying each condition by $(1+\beta)^{t+1}$. Moreover, the first-order condition for the optimal tax on capital income is derived for two separate cases: where the no-leverage constraint is slack and where it binds. If the constraint is slack, $\Delta_t = 0$ and the comparative statics (102) apply, so that the summation in eq. (97) reduces to its Domar-Musgrave term for $s = t$; if the constraint binds, $\Delta_t \geq 0$ and the comparative statics (103) apply. This gives:

$$\left.\frac{\partial \mathcal{L}}{\partial \tau_t^I}\right|_{\omega_t<1} = 0: \quad \mu_{t+1}\pi_t x_t + \mu_{t+1}\tau_t^I \pi_t \frac{x_t}{1-\tau_t^I} = \kappa_{t+1} \quad \Longleftrightarrow \quad \frac{\mu_{t+1}\pi_t x_t}{1-\tau_t^I} = \kappa_{t+1}, \tag{104}$$

$$\left.\frac{\partial \mathcal{L}}{\partial \tau_t^I}\right|_{\omega_t=1} = 0: \quad (\mu_{t+1}\pi_t - \Delta_t)x_t + \sum_{s=0}^{\infty} \frac{\mu_{s+1}}{(1+\beta)^{s-t}} \left[(1+\rho-\phi_s)\left.\frac{\partial s_s}{\partial \tau_t^I}\right|_{\phi_t} + \tau_s^I \pi_s \left.\frac{\partial x_s}{\partial \tau_t^I}\right|_{\phi_t}\right] = \kappa_{t+1}, \tag{105}$$

$$\frac{\partial \mathcal{L}}{\partial \phi_t} = 0: \quad (\mathrm{E}_t[\mathcal{V}'_{t+1}(\cdot)] - \mu_{t+1})s_t + \sum_{s=0}^{\infty} \frac{\mu_{s+1}}{(1+\beta)^{s-t}} \left[(1+\rho-\phi_s)\frac{\partial s_s}{\partial \phi_t} + \tau_s^I \pi_s \frac{\partial x_s}{\partial \phi_t}\right] = 0, \tag{106}$$

$$\frac{\partial \mathcal{L}}{\partial G_{t+1}} = 0: \quad \Gamma'(G_{t+1}) = \mu_{t+1}, \tag{107}$$

$$\frac{\partial \mathcal{L}}{\partial d_{t+1}} = 0: \quad \mu_t = \frac{(1+\rho)\mu_{t+1}}{(1+\beta)}, \tag{108}$$

$$\kappa_{t+1} \geq 0, \qquad 1-\tau_t^I \geq 0, \qquad \kappa_{t+1}(1-\tau_t^I) = 0. \tag{109}$$

Eqs. (104)–(109) are the system to be solved. This system is solved in a number of steps.

First, it is proven by contradiction that the no-leverage constraint binds at any optimum ($\omega_t = 1$). Suppose the no-leverage constraint on the period-t portfolio is slack. Then $\tau_t^I < 1$ (at $\tau_t^I = 1$ the individual selects $\omega_t = 1$, as argued below eq. (95)), so $\kappa_{t+1} = 0$ by eq. (101), and eq. (104) requires $\mu_{t+1}\pi_t x_t = 0$. But $\mu_{t+1} = \Gamma'(G_{t+1}) > 0$, $\pi_t > 0$ and $x_t > 0$ (with an interior

portfolio and $\pi_t > 0$, the individual always holds some equity, since at $\omega_t = 0$ we would have $\Delta_t = \pi_t \mathcal{V}'_{t+1} > 0$). This leads to a contradiction. Hence, we find that

$$\left.\frac{\partial \mathcal{L}}{\partial \tau_t^I}\right|_{\phi_t} = \frac{\mu_{t+1}\pi_t}{(1+\beta)^{t+1}} \frac{x_t}{(1-\tau_t^I)} > 0, \tag{110}$$

whenever the no-leverage constraint is slack. As a result, as long as $\omega_t < 1$, social welfare rises strictly with a higher tax on capital income – for a given net tax on the safe return ϕ_t. Thus, the government keeps on raising the tax on capital income. However, the Domar-Musgrave effect then implies that the portfolio share, $\omega_t = \omega_t^0(1-\tau_t^{I,0})/(1-\tau_t^I)$, keeps on rising too. Hence, ultimately, it reaches the corner of the no-leverage constraint $\omega_t = 1$ at the tax rate $\bar{\tau}_t \equiv 1 - \omega_t^0(1-\tau_t^{I,0}) < 1$. Therefore, at any optimum the no-leverage constraint binds ($\omega_t = 1$) for all t. This is the portfolio part of part 1 of Proposition 5.

Second, it is proven that optimal tax policy implies full taxation of excess returns, so that the tax on capital income is 100 percent ($\tau_t^I = 1$). For $\tau_t^I \in [\bar{\tau}_t, 1)$ first-order condition (105) applies with $\kappa_{t+1} = 0$. Its first term is bounded below using eq. (95):

$$x_t \left(\mu_{t+1}\pi_t - \Delta_t\right) \geq \pi_t x_t \left(\mu_{t+1} - \mathrm{E}_t[\mathcal{V}'_{t+1}(\cdot)]\right), \tag{111}$$

which holds with equality at $\tau_t^I = 1$. Hence, whenever the marginal value of public funds weakly exceeds the expected private marginal utility of next-period wealth ($\mu_{t+1} \geq \mathrm{E}_t[\mathcal{V}'_{t+1}]$) – and up to the wedge-weighted saving responses in the summation term, which vanish when all intertemporal wedges are zero – the left-hand side of eq. (105) is strictly positive for $\tau_t^I < 1$. That is, raising the tax on capital income – for given ϕ_t – continues to raise welfare at the corner solution for ω_t. As a result, full taxation of all capital incomes $\tau_t^I = 1$ is reached. At $\tau_s^I = 1$ and $x_s = s_s$ for all s, we have $\Delta_t = \pi_t u'(c_{t+1})$ and $\mathrm{E}_t[\mathcal{V}'_{t+1}] = u'(c_{t+1})$, so that the first-order condition for τ_t^I eq. (105) joint with the first-order condition for G_{t+1} in eq. (107) yields the multiplier κ_{t+1} on the upper bound:

$$\kappa_{t+1} = \pi_t s_t \left[\Gamma'(G_{t+1}) - u'(c_{t+1})\right] + \sum_{s=0}^{\infty} \frac{\Gamma'(G_{s+1})}{(1+\beta)^{s-t}} \mathcal{W}_s \left.\frac{\partial s_s}{\partial \tau_t^I}\right|_{\phi_t} \geq 0, \quad \mathcal{W}_s \equiv 1+\rho+\pi_s-\phi_s, \tag{112}$$

where $\mathcal{W}_s$ is the intertemporal net tax wedge on saving. With $\tau_s^I = 1$ and $x_s = s_s$, period-s revenue is $(1+\rho+\pi_s-\phi_s)s_s = \mathcal{W}_s s_s$. Eq. (112) confirms that the corner $\tau_t^I = 1$ is consistent with the Kuhn-Tucker conditions precisely when $\Gamma'(G_{t+1}) \geq u'(c_{t+1})$ (net of the wedge terms). This proves the capital income tax part of Proposition 5.

Third, the optimal intertemporal wedge on saving is derived. Impose $\tau_s^I = 1$, $x_s = s_s$ for all s in eq. (106). Under full insurance $\mathrm{E}_t[\mathcal{V}'_{t+1}] = u'(c_{t+1})$, and the bracket in the sum equals $\mathcal{W}_s \, \partial s_s/\partial \phi_t$. Substituting $\mu_{s+1} = \Gamma'(G_{s+1})$ from eq. (107) and dividing by $1+\beta$ yields:

$$\frac{s_t}{1+\beta}\left[u'(c_{t+1}) - \Gamma'(G_{t+1})\right] = -\sum_{s=0}^{\infty} \frac{\Gamma'(G_{s+1})}{(1+\beta)^{s-t+1}} \mathcal{W}_s \frac{\partial s_s}{\partial \phi_t}. \tag{113}$$

Under full insurance the individual's Euler equation (7) of the main text reduces to

$$u'(c_t) = \frac{\phi_t}{1+\beta} u'(c_{t+1}). \tag{114}$$

Eqs. (113), (107), (108) and (114) jointly determine the optimal paths of ϕ_t, G_t and d_t – if an optimum exists. Two cases are now discussed. Recall from the main text that at a zero wedge, $\mathcal{W}_s = 0$, the revenue from taxing the risk premium exactly finances the wealth subsidy, so that the tax system on capital raises no net revenue: whether the government can afford a zero wedge therefore depends on whether the exogenous lump-sum revenues T_t and the initial government position suffice to finance public goods.

First, the case where the exogenous resources are insufficient to finance public goods at a zero wedge. In that case, the government is forced to tax the normal return to saving at some date s: $\mathcal{W}_s > 0$ for some s. Provided that $\mathcal{W}_s \geq 0$ for all s and that saving does not decrease in the after-tax return ($\partial s_s / \partial \phi_t \geq 0$), the right-hand side of eq. (113) is negative, so that $\Gamma'(G_{t+1}) > u'(c_{t+1})$. Hence, the Samuelson rule is modified by the marginal cost of public funds, which captures the distortions arising from taxing the normal return to saving. As a result, the wealth subsidy is insufficient to eliminate all distortions from the 100 percent tax on capital income: $\phi_t < 1 + \rho + \pi_t$, i.e. $\tau_t^A > -(\rho + \pi_t)$. This gives the rule for optimal provision of public goods and the optimal tax on wealth.

Second, the case where the exogenous resources are sufficient to finance public goods at a zero wedge. In that case, the intertemporal wedge on saving is zero for all s: $\mathcal{W}_s = 0$, so that $\phi_s = 1 + \rho + \pi_s$. The right-hand side of eq. (113) vanishes and its left-hand side gives the period-by-period Samuelson rule $\Gamma'(G_{t+1}) = u'(c_{t+1})$ for all $t \geq 0$. Public goods provision is now efficient.

However, $\phi_s = 1 + \rho + \pi_s$ should also be consistent with the government's Euler equation, i.e. the first-order condition for government debt (108), and the individual Euler equation (114). For any $t \geq 1$, substituting $\Gamma'(G_t) = u'(c_t)$ and $\Gamma'(G_{t+1}) = u'(c_{t+1})$ into eq. (108) and comparing with eq. (114) gives:

$$\underbrace{\frac{u'(c_t)}{u'(c_{t+1})} = \frac{1+\rho}{1+\beta}}_{\text{government Euler}} \quad \text{versus} \quad \underbrace{\frac{u'(c_t)}{u'(c_{t+1})} = \frac{1+\rho+\pi_t}{1+\beta}}_{\text{private Euler}}. \tag{115}$$

The government and private sector Euler equations cannot simultaneously hold unless the risk premium is zero: $\pi_t = 0$. Hence, for a positive risk premium $\pi_t > 0$, the first-order-condition system (107), (108), (113), (114) has *no* solution with a zero intertemporal wedge $\mathcal{W}_s = 0$ for all s. The first-order conditions for the optimal taxes on capital income τ_t^I, the net after-tax returns ϕ_t and optimal public goods provision G_t can all be satisfied, but the solution violates the optimal debt condition. Hence, this is not an optimal policy solution.

H.1 Optimal taxation with idiosyncratic risk and deterministic lump-sum taxes

This sub-appendix asks whether the non-existence result derived above is an artefact of treating the deterministic lump-sum tax T_t as exogenous. If the government is forced to finance public goods with distortionary instruments, it may seem natural that the first best cannot be attained. It is shown here that this intuition is incorrect. Adding T_t to the government's instrument set does not restore the first best. Instead, the optimal-tax problem ceases to have a solution altogether.

The government's instruments are now $\{\tau_t^I, \phi_t, T_t, G_{t+1}, d_{t+2}\}$ for all $t \geq 0$. Everything else is as in the main text: policy is deterministic, all aggregates are deterministic by the exact law of large numbers, and the expectation E_0 over individual histories is suppressed. The Lagrangian is unchanged – deterministic lump-sum taxes already appear in eq. (96) – and is now differentiated with respect to T_t as well. The first-order conditions (97)–(101) continue to hold, and the additional first-order condition is given by:

$$\frac{\partial \mathcal{L}}{\partial T_t} = 0: \quad \frac{\partial \mathcal{V}_0}{\partial T_t} + \frac{\mu_{t+1}}{(1+\beta)^{t+1}} + \sum_{s=0}^{\infty} \frac{\mu_{s+1}}{(1+\beta)^{s+1}} \left[(1+\rho-\phi_s) \frac{\partial s_s}{\partial T_t} + \tau_s^I \pi_s \frac{\partial x_s}{\partial T_t} \right] = 0. \tag{116}$$

Since policy is deterministic, the dynamic Roy identity (22) applied to eq. (21) – equivalently, eq. (26) without the density factor – gives $\partial \mathcal{V}_0 / \partial T_t = -(1+\beta)^{-(t+1)} u'(c_{t+1})$. Substituting and multiplying eq. (116) by $(1+\beta)^{t+1}$ gives:

$$\mu_{t+1} - u'(c_{t+1}) + \sum_{s=0}^{\infty} \frac{\mu_{s+1}}{(1+\beta)^{s-t}} \left[(1+\rho-\phi_s) \frac{\partial s_s}{\partial T_t} + \tau_s^I \pi_s \frac{\partial x_s}{\partial T_t} \right] = 0. \tag{117}$$

First, the two corner results derived above are unaffected. The lump-sum tax does not enter the first-order condition for the tax on capital income, and it does not change the comparative statics (102) and (103) of the compensated reform $\mathrm{d}\tau_t^A = -\rho \mathrm{d}\tau_t^I$, which holds T_t fixed by construction. Hence, eq. (104) still implies that the no-leverage constraint binds at any optimum ($\omega_t = 1$), and eq. (105) still implies that the tax on capital income is driven to its upper bound ($\tau_t^I = 1$). At $\tau_s^I = 1$ and $x_s = s_s$ for all s, individuals are fully insured, so that $\mathrm{E}_t[\mathcal{V}'_{t+1}] = u'(c_{t+1})$, and the bracket in every behavioural sum collapses to the intertemporal net tax wedge on saving:

$$(1+\rho-\phi_s) \frac{\partial s_s}{\partial z_t} + \pi_s \frac{\partial s_s}{\partial z_t} = \mathcal{W}_s \frac{\partial s_s}{\partial z_t}, \qquad \mathcal{W}_s \equiv 1 + \rho + \pi_s - \phi_s, \tag{118}$$

for each instrument $z_t \in \{\phi_t, T_t\}$. The wedge therefore reappears inside the very first-order condition that was supposed to eliminate it.

Second, the first-order conditions for ϕ_t and T_t are combined. Imposing $\tau_s^I = 1$ and $x_s = s_s$

in eqs. (106) and (117) and using eq. (118) gives:

$$\frac{\partial \mathcal{L}}{\partial \phi_t} = 0: \quad \big(u'(c_{t+1}) - \mu_{t+1}\big)\, s_t + \sum_{s=0}^{\infty} \frac{\mu_{s+1}}{(1+\beta)^{s-t}} \mathcal{W}_s \frac{\partial s_s}{\partial \phi_t} = 0, \tag{119}$$

$$\frac{\partial \mathcal{L}}{\partial T_t} = 0: \quad \big(\mu_{t+1} - u'(c_{t+1})\big) + \sum_{s=0}^{\infty} \frac{\mu_{s+1}}{(1+\beta)^{s-t}} \mathcal{W}_s \frac{\partial s_s}{\partial T_t} = 0. \tag{120}$$

Multiplying eq. (120) by s_t and adding it to eq. (119) makes the direct utility effect and the mechanical revenue effect cancel exactly, so that only the behavioural terms remain:

$$\sum_{s=0}^{\infty} \frac{\mu_{s+1}}{(1+\beta)^{s-t}} \mathcal{W}_s \left[\frac{\partial s_s}{\partial \phi_t} + s_t \frac{\partial s_s}{\partial T_t} \right] = 0, \qquad \forall t \geq 0. \tag{121}$$

The bracket is the *compensated* response of period-s saving to the after-tax safe return in period t.[37] Writing $\tilde{s}_s$ for compensated saving, the Slutsky decomposition is $\partial \tilde{s}_s / \partial \phi_t = \partial s_s / \partial \phi_t + s_t \, \partial s_s / \partial T_t$, so that eq. (121) reads:

$$\sum_{s=0}^{\infty} \frac{\mu_{s+1}}{(1+\beta)^{s-t}} \mathcal{W}_s \frac{\partial \tilde{s}_s}{\partial \phi_t} = 0, \qquad \forall t \geq 0. \tag{122}$$

This is the standard optimal Ramsey tax structure: with a non-distortionary instrument available, only compensated behavioural responses matter, and the income effects in eq. (113) vanish. Eq. (122) is a linear system in the vector of wedges $(\mathcal{W}_s)_{s \geq 0}$ with coefficient matrix $\mu_{s+1}(1+\beta)^{t-s} \partial \tilde{s}_s / \partial \phi_t$. Since $\mu_{s+1} = \Gamma'(G_{s+1}) > 0$ and the discount factors are strictly positive, this matrix is non-singular whenever the matrix of compensated saving responses is non-singular, which is the standard regularity condition.[38] Hence:

$$\mathcal{W}_s = 0 \quad \forall s \qquad \Longleftrightarrow \qquad \phi_s = 1 + \rho + \pi_s \quad \forall s. \tag{123}$$

This is the expected result: once a non-distortionary instrument is available, the government eliminates every distortionary wedge and finances public goods with lump-sum taxes. The tax system on capital then raises no net revenue at all, since period-s revenue equals $\mathcal{W}_s s_s = 0$: the 100 percent tax on capital income raises $\pi_s s_s$ and the wealth subsidy $\tau_s^A = -(\rho + \pi_s)$ returns exactly the same amount.

Third, the system is shown to be over-determined. With $\mathcal{W}_s = 0$ the behavioural sums in eqs. (119) and (120) vanish, so that these conditions reduce to $\mu_{t+1} = u'(c_{t+1})$. Combining this with eq. (107) yields the period-by-period Samuelson rule $\Gamma'(G_{t+1}) = u'(c_{t+1})$ for all $t \geq 0$, so that public goods are efficiently provided. Note that eq. (119) is then satisfied *identically*: the direct utility effect and the mechanical revenue effect of ϕ_t cancel exactly, as in the case with

[37] To see this, note that ϕ_t and T_t enter period-$t+1$ assets as $a_{t+1} = \phi_t s_t - T_t + \ldots$, so that $\partial a_{t+1} / \partial \phi_t = s_t$ and $\partial a_{t+1} / \partial T_t = -1$. A change $\mathrm{d}\phi_t$ accompanied by $\mathrm{d}T_t = s_t \mathrm{d}\phi_t$ therefore leaves period-$t+1$ resources unchanged at the initial level of saving, which corresponds to the compensated response.

[38] The matrix of compensated responses is symmetric and negative semi-definite in the implicit present-value prices $q_t = \prod_{\tau < t} \phi_\tau^{-1}$ and has the usual one-dimensional null space corresponding to an equiproportional change in all prices. That null direction is a uniform tax on all future consumption, which is not attainable here, since period-0 consumption $c_0 = a_0 - s_0$ cannot be taxed. Restricted to the attainable directions, the matrix is negative definite.

aggregate risk and state-contingent lump-sum taxes, so that ϕ_t is no longer pinned down by its own first-order condition, but only through eq. (123). Substituting the Samuelson rule into the debt condition (108) and comparing with the individual Euler equation (114) at $\phi_t = 1+\rho+\pi_t$ reproduces eq. (115) exactly. The government and private-sector Euler equations cannot hold simultaneously unless $\pi_t = 0$. Hence, the system of first-order-conditions for optimal policy has again no solution.

Fourth, and in contrast to the case with exogenous lump-sum taxes, there is now no well-defined second-best case, since the government can raise revenue without distortions. Hence, it never wants a positive intertemporal wedge, and eq. (123) applies. Making the lump-sum tax a choice variable therefore removes the only case in which the optimal-tax problem has a solution.

The over-determination is caused by the *joint* presence of a lump-sum tax and public debt, and not by either of them separately. With public debt but no lump-sum instrument – the case of Proposition 5 with insufficient exogenous resources – the government is forced to set a positive intertemporal wedge and a second-best optimum exists. With a lump-sum tax but no public debt, the first-order condition (108) disappears, so that $\mu_t = \frac{1+\rho}{1+\beta}\mu_{t+1}$ does not need to hold. Hence, the remaining conditions $\mu_{t+1} = u'(c_{t+1}) = \Gamma'(G_{t+1})$ and $\mathcal{W}_s = 0$ are mutually consistent for any path of risk premia, the government balances its budget period by period, and first best *is* attained. Therefore, only the combination of the two leads to a breakdown of the optimality conditions.

The reason why a lump-sum instrument cannot repair the problem is that the conflict concerns the *pricing* of resources over time and not the *raising* of revenue. A lump-sum tax gives a non-distortionary instrument to allocate resources between the private and the public sector, but it does not change the rate at which either sector transforms resources over time. At $\tau_t^I = 1$ with a binding no-leverage constraint, the private sector transforms resources at ϕ_t, the government transforms resources at $1+\rho$ through public debt, and the economy transforms resources at $1+\rho+\pi_t$, since all saving is invested in equity. Efficiency requires $\phi_t = 1+\rho+\pi_t$, while the first-order condition for debt requires $\phi_t = 1+\rho$. A lump-sum tax does not affect any of these conditions, but it does eliminate the only second-best optimal policy solution that is well-defined.

I Proof Proposition 6

The government's instruments are $\{\tau_t^I, \phi_t, T_t(\theta_t), G_{t+1}(\theta_{t+1}), d_{t+2}\}$ for all $t \geq 0$, with $\phi_t \equiv 1+(1-\tau_t^I)\rho-\tau_t^A$. The state-contingent lump-sum taxes are now optimised alongside the taxes on capital income and wealth, public goods and debt. All instruments are history-contingent plans: the tax rates τ_t^I and ϕ_t on period-t saving are set before θ_t is realised and are therefore measurable with respect to θ^{t-1}, like the individual's period-t decisions; the lump-sum tax on period-t saving, $T_t(\theta_t)$, is set after θ_t is realised and is measurable with respect to θ^t. Period-$t+1$ public goods $G_{t+1}(\theta_{t+1})$ and debt d_{t+2} are set after θ_t – and, in general, θ_{t+1} – is realised and are measurable with respect to θ^{t+1}. As in the main text, $\mathrm{E}_t[\cdot]$ denotes the expectation conditional on θ^{t-1}, so that $\mathrm{E}_{t+1}[\cdot]$ conditions on θ^t. As in the main text, $f(\theta_t)$ is shorthand

for the density $f(\theta_t \mid \theta^{t-1})$ of θ_t conditional on θ^{t-1} – the density against which $\mathrm{E}_t[\cdot]$ integrates – and $f(\theta^t) = f(\theta^{t-1})f(\theta_t)$ is the density of the history θ^t. The dependence of all variables on the state history is suppressed. Saving s_t, the portfolio share ω_t and the gross equity exposure $x_t \equiv \omega_t s_t$ are functions of the entire path of all government instruments. Throughout, the no-leverage constraint is slack ($\omega_t < 1$), as assumed in the main text, and the optimal tax on capital income turns out to be interior ($\tau_t^I < 1$), so that no inequality constraints need to be taken into account. Since the portfolio is interior, the individual's portfolio condition (8) holds with equality:

$$\Delta_t \equiv \mathrm{E}_t[\mathcal{V}'_{t+1}(\cdot)(r_t(\theta_t) - \rho)] = 0. \tag{124}$$

Using the reparametrisation, the government budget constraint (12) of period $t+1$ can be solved for public goods and written in terms of tax wedges:

$$G_{t+1}(\theta_{t+1}) = (1 + \rho - \phi_t)s_t + \tau_t^I(r_t(\theta_t) - \rho)x_t + T_t(\theta_t) + d_{t+2} - (1+\rho)d_{t+1}, \tag{125}$$

where the first term is revenue from the intertemporal wedge $1+\rho-\phi_t$ on saving and the second term is stochastic revenue from taxing the risk premium on equity. The optimal tax system is solved by maximising social welfare $\mathcal{V}_0(a_0)$ subject to eq. (125) for every period and history and the no-Ponzi condition. Attach stochastic Lagrange multiplier $(1+\beta)^{-(t+1)}\mu_{t+1}$ to the period-$t+1$ budget constraint in each history, where μ_{t+1} – the marginal social value of public resources in period-$t+1$ utility units – is measurable with respect to θ^{t+1}. The Lagrangian is then given by:

$$\begin{aligned}\mathcal{L} \;=\; \mathcal{V}_0(a_0) + \sum_{t=0}^{\infty} \frac{1}{(1+\beta)^{t+1}} \mathrm{E}_0\Big[\mu_{t+1}\Big((1+\rho-\phi_t)s_t + \tau_t^I(r_t(\theta_t)-\rho)x_t + T_t(\theta_t) \\ + d_{t+2} - (1+\rho)d_{t+1} - G_{t+1}(\theta_{t+1})\Big)\Big].\end{aligned} \tag{126}$$

The necessary first-order conditions are found by differentiating eq. (126) with respect to the value of each policy instrument at each history at which it is set. Since $\mathcal{V}_0(a_0)$ and the budget terms are expectations over histories, each such derivative is proportional to the density of that history. The direct welfare effect has the density through Roy's identity – cf. Lemma (2) – and the mechanical revenue effect features the same density times the expectation of the

multiplier conditional on the history. The first-order conditions are thus given by:

$$\frac{\partial \mathcal{L}}{\partial \tau_t^I} = 0: \quad \left.\frac{\partial \mathcal{V}_0}{\partial \tau_t^I}\right|_{\phi_t} + \frac{f(\theta^{t-1}) x_t \mathrm{E}_t[\mu_{t+1}(r_t(\theta_t) - \rho)]}{(1+\beta)^{t+1}} + \sum_{s=0}^{\infty} \frac{\mathrm{E}_0\left[\mu_{s+1}\left((1+\rho-\phi_s)\left.\frac{\partial s_s}{\partial \tau_t^I}\right|_{\phi_t} + \tau_s^I(r_s(\theta_s)-\rho)\left.\frac{\partial x_s}{\partial \tau_t^I}\right|_{\phi_t}\right)\right]}{(1+\beta)^{s+1}} = 0, \tag{127}$$

$$\frac{\partial \mathcal{L}}{\partial \phi_t} = 0: \quad \frac{\partial \mathcal{V}_0}{\partial \phi_t} - \frac{f(\theta^{t-1}) s_t \mathrm{E}_t[\mu_{t+1}]}{(1+\beta)^{t+1}} + \sum_{s=0}^{\infty} \frac{\mathrm{E}_0\left[\mu_{s+1}\left((1+\rho-\phi_s)\frac{\partial s_s}{\partial \phi_t} + \tau_s^I(r_s(\theta_s)-\rho)\frac{\partial x_s}{\partial \phi_t}\right)\right]}{(1+\beta)^{s+1}} = 0, \tag{128}$$

$$\frac{\partial \mathcal{L}}{\partial T_t(\theta_t)} = 0: \quad \frac{\partial \mathcal{V}_0}{\partial T_t(\theta_t)} + \frac{f(\theta^t) \mathrm{E}_{t+1}[\mu_{t+1}]}{(1+\beta)^{t+1}} + \sum_{s=0}^{\infty} \frac{\mathrm{E}_0\left[\mu_{s+1}\left((1+\rho-\phi_s)\frac{\partial s_s}{\partial T_t(\theta_t)} + \tau_s^I(r_s(\theta_s)-\rho)\frac{\partial x_s}{\partial T_t(\theta_t)}\right)\right]}{(1+\beta)^{s+1}} = 0, \tag{129}$$

$$\frac{\partial \mathcal{L}}{\partial G_{t+1}(\theta_{t+1})} = 0: \quad \frac{\partial \mathcal{V}_0}{\partial G_{t+1}(\theta_{t+1})} - \frac{f(\theta^{t+1})\mu_{t+1}}{(1+\beta)^{t+1}} = 0, \tag{130}$$

$$\frac{\partial \mathcal{L}}{\partial d_{t+1}} = 0: \quad \frac{f(\theta^t)\mu_t}{(1+\beta)^t} - \frac{f(\theta^t)(1+\rho)\mathrm{E}_{t+1}[\mu_{t+1}]}{(1+\beta)^{t+1}} = 0, \tag{131}$$

where the following properties have been used: (i) since public goods are additively separable, s_s and x_s do not depend on G or d, but – since lump-sum taxes are state-contingent – they do depend on the entire path of lump-sum taxes: a change in $T_t(\theta_t)$ in one state changes both the level and the state-contingency of next-period assets and thereby moves saving and exposure in all periods. The mechanical revenue effect of $T_t(\theta_t)$ in eq. (129) is the density of the history θ^t times the expected marginal value of period-$t+1$ public resources given that history, $\mathrm{E}_{t+1}[\mu_{t+1}]$; (ii) since (τ_t^I, ϕ_t) is a bijective reparametrisation of (τ_t^I, τ_t^A), the derivatives with respect to τ_t^I in eq. (127) are taken at a fixed ϕ_t (i.e. jointly with $\mathrm{d}\tau_t^A = -\rho \mathrm{d}\tau_t^I$); the tax rates are set in history θ^{t-1}, so that their mechanical revenue effects carry the density $f(\theta^{t-1})$ and the expectation $\mathrm{E}_t[\cdot]$ over θ_t; (iii) in eq. (130), $G_{t+1}(\theta_{t+1})$ is set in history θ^{t+1}, in which μ_{t+1} is known; (iv) in eq. (131), d_{t+1} is set in history θ^t and appears with coefficient $+1$ in the period-t constraint (multiplier μ_t, known in θ^t) and with coefficient $-(1+\rho)$ in the period-$t+1$ constraint (multiplier μ_{t+1}, of which only the conditional expectation $\mathrm{E}_{t+1}[\mu_{t+1}]$ given θ^t is known when d_{t+1} is set).

To simplify the first-order conditions, Lemma (2) is applied: the dynamic Roy identity (22) and the period-t partial derivatives (18), (19), (20) and (21). Eq. (19) is zero by eq. (124), since a marginal increase in the tax on the excess return has no first-order utility cost if the after-tax safe return is constant. Eq. (18) simplifies to $u'(c_t)s_t/\phi_t$ for the same reason. For the state-contingent instruments, the direct welfare effects are eqs. (26) and (25), the latter applied at $t+1$: $\partial \mathcal{V}_0/\partial T_t(\theta_t) = -(1+\beta)^{-(t+1)} u'(c_{t+1}(\theta_t)) f(\theta^t)$ and $\partial \mathcal{V}_0/\partial G_{t+1}(\theta_{t+1}) = (1+\beta)^{-(t+1)}\Gamma'(G_{t+1}(\theta_{t+1}))f(\theta^{t+1})$. Each has the density of the history at which the instrument is set, as do the corresponding mechanical revenue effects in eqs. (130) and (129). Public debt has no direct effect on $\mathcal{V}_t$.

Further, under the particular policy parameterisation, the comparative statics are as follows.

The compensated perturbation $\mathrm{d}\tau_t^I$ at fixed ϕ_t – with the lump-sum taxes held fixed – leaves the individual's opportunity set unchanged (the after-tax safe return is fixed, and any after-tax exposure remains attainable through ω_t), so that the entire allocation is unchanged and only the portfolio share adjusts:

$$\left.\frac{\partial s_s}{\partial \tau_t^I}\right|_{\phi_t} = 0 \;\forall s, \quad \left.\frac{\partial x_t}{\partial \tau_t^I}\right|_{\phi_t} = \frac{x_t}{1-\tau_t^I}, \quad \left.\frac{\partial x_s}{\partial \tau_t^I}\right|_{\phi_t} = 0 \;\forall s \neq t. \tag{132}$$

This is just a replication of Lemma (3) in the main text. The responses to ϕ_t and to $T_t(\theta_t)$, $\partial s_s/\partial \phi_t$, $\partial s_s/\partial T_t(\theta_t)$ and $\partial x_s/\partial \phi_t$, $\partial x_s/\partial T_t(\theta_t)$, are uncompensated and generally nonzero in all periods s. They never need to be derived explicitly.

The first-order conditions can be simplified by substituting eqs. (18)–(21) and the dynamic Roy identity (22) into eqs. (127)–(131), using the comparative statics (132) in eq. (127) – so that its summation reduces to the Domar-Musgrave term for $s = t$ – and multiplying each condition by $(1+\beta)^{t+1}$. In the mechanical terms, the law of iterated expectations, $\mathrm{E}_t[X] = \mathrm{E}_t[\mathrm{E}_{t+1}[X]]$ for any random variable X (sometimes called the tower property of conditional expectations), gives $\mathrm{E}_t[\mu_{t+1}(r_t(\theta_t)-\rho)] = \mathrm{E}_t[\mathrm{E}_{t+1}[\mu_{t+1}](r_t(\theta_t)-\rho)]$ and $\mathrm{E}_t[\mu_{t+1}] = \mathrm{E}_t[\mathrm{E}_{t+1}[\mu_{t+1}]]$, since $r_t(\theta_t)$ is known once θ^t is known. This gives:

$$\frac{\partial \mathcal{L}}{\partial \tau_t^I} = 0: \quad \frac{f(\theta^{t-1})x_t}{1-\tau_t^I}\mathrm{E}_t\left[\mathrm{E}_{t+1}[\mu_{t+1}](r_t(\theta_t)-\rho)\right] = 0, \tag{133}$$

$$\frac{\partial \mathcal{L}}{\partial \phi_t} = 0: \quad f(\theta^{t-1})\left(\mathrm{E}_t[\mathcal{V}'_{t+1}(\cdot)] - \mathrm{E}_t[\mathrm{E}_{t+1}[\mu_{t+1}]]\right)s_t \tag{134}$$
$$+\sum_{s=0}^{\infty}\frac{\mathrm{E}_0\left[\mu_{s+1}\left((1+\rho-\phi_s)\frac{\partial s_s}{\partial \phi_t} + \tau_s^I(r_s(\theta_s)-\rho)\frac{\partial x_s}{\partial \phi_t}\right)\right]}{(1+\beta)^{s-t}} = 0,$$

$$\frac{\partial \mathcal{L}}{\partial T_t(\theta_t)} = 0: \quad f(\theta^t)\left(\mathrm{E}_{t+1}[\mu_{t+1}] - u'(c_{t+1}(\theta_t))\right) \tag{135}$$
$$+\sum_{s=0}^{\infty}\frac{\mathrm{E}_0\left[\mu_{s+1}\left((1+\rho-\phi_s)\frac{\partial s_s}{\partial T_t(\theta_t)} + \tau_s^I(r_s(\theta_s)-\rho)\frac{\partial x_s}{\partial T_t(\theta_t)}\right)\right]}{(1+\beta)^{s-t}} = 0,$$

$$\frac{\partial \mathcal{L}}{\partial G_{t+1}(\theta_{t+1})} = 0: \quad f(\theta^{t+1})\left(\Gamma'(G_{t+1}(\theta_{t+1})) - \mu_{t+1}\right) = 0, \tag{136}$$

$$\frac{\partial \mathcal{L}}{\partial d_{t+1}} = 0: \quad f(\theta^t)\left(\mu_t - \frac{1+\rho}{1+\beta}\mathrm{E}_{t+1}[\mu_{t+1}]\right) = 0. \tag{137}$$

Eqs. (133)–(137) are the system to be solved. All densities are strictly positive. In eqs. (133), (136) and (137) the density multiplies the entire condition, so that it can be divided out immediately: these conditions hold history by history. In eqs. (134) and (135) the density only multiplies the direct and mechanical terms, whereas the behavioural sums are unconditional expectations $\mathrm{E}_0[\cdot]$ that integrate over all histories. These two conditions can therefore only be divided by the density after the behavioural sums have been shown to vanish, which is done below.

This system is solved by verifying that the following solution candidate satisfies every first-order condition and attains the first best: $\phi_s = 1+\rho$ for all s; an arbitrary $\tau_s^I \in [0,1)$; lump-sum taxes such that public goods satisfy the Samuelson rule state by state (eq. (141) below); and

public goods and debt satisfying eqs. (136) and (137).

First, the multipliers are eliminated. Dividing eq. (136) by the density $f(\theta^{t+1})$ gives $\mu_{t+1} = \Gamma'(G_{t+1}(\theta_{t+1}))$ in every history; dividing eq. (137) by $f(\theta^t)$ and substituting then gives the government's Euler equation:

$$\Gamma'(G_t(\theta_t)) = \frac{1+\rho}{1+\beta}\mathrm{E}_{t+1}[\Gamma'(G_{t+1}(\theta_{t+1}))], \tag{138}$$

which is the stochastic counterpart of the individual's Euler equation: the government smooths the marginal value of public funds over time at the safe rate. Its implication that is used below is

$$\mathrm{E}_{t+1}[\mu_{t+1}] = \frac{1+\beta}{1+\rho}\Gamma'(G_t(\theta_t)). \tag{139}$$

The expected marginal value of period-$t+1$ public resources, given θ^t, equals the marginal utility of period-t public goods, corrected for the one-period difference in timing.

Second, all behavioural terms are shown to vanish at the proposed solution. The intertemporal wedges $1+\rho-\phi_s$ are zero, so all saving responses in eqs. (134) and (135) drop out. For the exposure responses, note that at the candidate lump-sum taxes – eq. (141) below – $\Gamma'(G_s(\theta_s))$ is proportional to $u'(c_{s+1}(\theta_s))$ state by state, so that by eq. (139)

$$\begin{aligned} \mathrm{E}_s\left[\mathrm{E}_{s+1}[\mu_{s+1}](r_s(\theta_s)-\rho)\right] &= \frac{1+\beta}{1+\rho}\mathrm{E}_s[\Gamma'(G_s(\theta_s))(r_s(\theta_s)-\rho)] \\ &= \mathrm{E}_s[u'(c_{s+1})(r_s(\theta_s)-\rho)] = \Delta_s = 0, \end{aligned} \tag{140}$$

where the last equality is the individual's portfolio condition (124), which applies since $\tau_s^I < 1$. Since $\partial x_s/\partial\phi_t$ and $\partial x_s/\partial T_t(\theta_t)$ are measurable with respect to θ^{s-1}, the law of iterated expectations gives, for example, $\mathrm{E}_0[\mu_{s+1}(r_s(\theta_s)-\rho)\partial x_s/\partial\phi_t] = \mathrm{E}_0[(\partial x_s/\partial\phi_t)\mathrm{E}_s[\mathrm{E}_{s+1}[\mu_{s+1}](r_s(\theta_s)-\rho)]] = 0$, so that every exposure term in eqs. (134) and (135) vanishes. With the behavioural sums gone, the densities in eqs. (133), (134) and (135) can be eliminated.

Third, the optimal lump-sum taxes are derived. Eq. (135) now reduces to $\mathrm{E}_{t+1}[\mu_{t+1}] = u'(c_{t+1}(\theta_t))$ in every history θ^t, and substituting eq. (139) yields the Samuelson rule for every state θ_t:

$$\Gamma'(G_t(\theta_t)) = \frac{1+\rho}{1+\beta}u'(c_{t+1}(\theta_t)). \tag{141}$$

The government equates the marginal value of public funds to private marginal utility in every state separately, because the state-contingent lump-sum tax transfers resources between the private and the public sector state by state without any distortion. Taking expectations over θ_t and using the Euler equation (7) at the candidate $\phi_t = 1+\rho$, i.e. $\mathrm{E}_t[u'(c_{t+1})] = \frac{1+\beta}{1+\rho}u'(c_t)$, delivers the Samuelson rule in expectation: $\mathrm{E}_t[\Gamma'(G_t(\theta_t))] = u'(c_t)$.

Fourth, the first-order condition for the tax on capital income holds for every $\tau_t^I \in [0,1)$. By eq. (140), the left-hand side of eq. (133) is identically zero at the proposed solution: once the lump-sum taxes align $\Gamma'(G_t(\theta_t))$ with $u'(c_{t+1}(\theta_t))$ state by state, the government's risk-allocation condition coincides with the individual's portfolio condition. The first-order condition for the tax on capital income therefore does not pin down τ_t^I: it holds identically for all combinations where $\tau_t^A = -\tau_t^I\rho$. This is the first-order-condition form of the indeterminacy in Proposition 6.

By the argument of Proposition 3, every $\tau_t^I \in [0,1)$ combined with $\tau_t^A = -\tau_t^I \rho$ and re-optimised lump-sum taxes implements the same allocation. The degenerate endpoint $\tau_t^I = 1$ is excluded: individuals then bear no excess-return risk for any portfolio composition, ω_t is indeterminate, and the portfolio condition (124) is vacuous.

Fifth, the first-order condition for the after-tax safe return is verified. Eq. (134) reduces to $\mathrm{E}_t[\mathcal{V}'_{t+1}] = \mathrm{E}_t[\mathrm{E}_{t+1}[\mu_{t+1}]]$, i.e. $\mathrm{E}_t[u'(c_{t+1})] = \mathrm{E}_t[u'(c_{t+1})]$ by eq. (135) – which holds identically. Hence, with optimal state-contingent lump-sum taxes, the direct utility effect and the mechanical revenue effect of ϕ_t cancel exactly for any ϕ_t, and eq. (134) is satisfied if and only if the saving wedges vanish, which is the case at $\phi_s = 1+\rho$ for all s. Any $\phi_s \neq 1+\rho$ would distort the intertemporal margin while a non-distorting instrument is available and is therefore dominated. The after-tax safe return is uniquely pinned down at $\phi_t = 1+\rho$. Since $\phi_t \equiv 1+(1-\tau_t^I)\rho-\tau_t^A$, this is equivalent to $\tau_t^A = -\tau_t^I \rho$, and, for the ex post wealth tax, $\phi_t \equiv (1-\tau_t^P)(1+(1-\tau_t^I)\rho) = 1+\rho$ is equivalent to $\tau_t^P = -\tau_t^I \rho/(1+(1-\tau_t^I)\rho)$.

Finally, the candidate attains the first best. The intertemporal margin is undistorted ($\phi_t = 1+\rho$, so that the individual's Euler equation coincides with the government's Euler equation (138) in expectation). The division of resources between private and public consumption is efficient state by state, by eq. (141). Aggregate risk is allocated efficiently, since private and public marginal valuations of the excess return are both zero, by eqs. (124) and (140). The government finances its budget entirely with instruments that distort no margin, so no feasible tax policy can attain higher welfare. Hence, the proposed solution is the global optimum. This proves Proposition 6.

J Proof Proposition 7

The government's instruments are $\{\tau_t^I, \phi_t, G_{t+1}(\theta_{t+1}), d_{t+2}\}$ for all $t \geq 0$, with $\phi_t \equiv 1+(1-\tau_t^I)\rho-\tau_t^A$. The state-contingent lump-sum taxes $T_t(\theta_t)$ are exogenously fixed, so that state-contingent public goods provision absorbs the stochastic revenue. All instruments are history-contingent plans: the tax rates τ_t^I and ϕ_t on period-t saving are set before θ_t is realised and are therefore measurable with respect to θ^{t-1}, like the individual's period-t decisions; period-$t+1$ public goods $G_{t+1}(\theta_{t+1})$ and debt d_{t+2} are set after θ_t – and, in general, θ_{t+1} – is realised and are measurable with respect to θ^{t+1}. As in the main text, $\mathrm{E}_t[\cdot]$ denotes the expectation conditional on θ^{t-1}, so that $\mathrm{E}_{t+1}[\cdot]$ conditions on θ^t. As in the main text, $f(\theta_t)$ is shorthand for the density $f(\theta_t \mid \theta^{t-1})$ of θ_t conditional on θ^{t-1} – the density against which $\mathrm{E}_t[\cdot]$ integrates – and $f(\theta^t) = f(\theta^{t-1})f(\theta_t)$ is the density of the history θ^t. The dependence of all variables on the state history is suppressed. Saving s_t, the portfolio share ω_t and the gross equity exposure $x_t \equiv \omega_t s_t$ are functions of the entire path of all policy instruments. Throughout, the no-leverage constraint is slack ($\omega_t < 1$), as assumed in the main text, and the optimal tax on capital income turns out to be interior ($\tau_t^I < 1$), so that no inequality constraints need to be taken into account. Since the portfolio is interior, the individual's portfolio condition (8) holds with equality:

$$\Delta_t \equiv \mathrm{E}_t[\mathcal{V}'_{t+1}(\cdot)(r_t(\theta_t) - \rho)] = 0. \tag{142}$$

Using the reparametrisation, the government budget constraint (12) of period $t+1$ can be

solved for public goods and written in terms of tax wedges:

$$G_{t+1}(\theta_{t+1}) = (1+\rho-\phi_t)s_t + \tau_t^I(r_t(\theta_t)-\rho)x_t + T_t(\theta_t) + d_{t+2} - (1+\rho)d_{t+1}, \tag{143}$$

where the first term is revenue from the intertemporal wedge $1+\rho-\phi_t$ on saving and the second term is the – now stochastic – revenue from taxing the risk premium on equity. The optimal tax system is solved by maximising social welfare $\mathcal{V}_0(a_0)$ subject to eq. (143) for every period and history and the no-Ponzi condition. Attach stochastic Lagrange multiplier $(1+\beta)^{-(t+1)}\mu_{t+1}$ to the period-$t+1$ budget constraint in each history, where μ_{t+1} – the marginal social value of public resources in period-t+1 utility units – is measurable with respect to θ^{t+1}. The Lagrangian is then given by:

$$\begin{aligned}\mathcal{L} \;=\; \mathcal{V}_0(a_0) + \sum_{t=0}^{\infty}\frac{1}{(1+\beta)^{t+1}}\mathrm{E}_0\Big[\mu_{t+1}\Big((1+\rho-\phi_t)s_t + \tau_t^I(r_t(\theta_t)-\rho)x_t + T_t(\theta_t) \\ +d_{t+2} - (1+\rho)d_{t+1} - G_{t+1}(\theta_{t+1})\Big)\Big].\end{aligned} \tag{144}$$

The necessary first-order conditions are found by differentiating eq. (144) with respect to the value of each instrument at each history at which it is set. Since $\mathcal{V}_0(a_0)$ and the budget terms are expectations over histories, each such derivative is proportional to the density of that history: the direct welfare effect carries the density through Roy's identity – cf. Lemma (2) – and the mechanical revenue effect carries the same density times the expectation of the multiplier conditional on the history. The first-order conditions are:

$$\begin{aligned}\frac{\partial\mathcal{L}}{\partial\tau_t^I} = 0: \quad & \frac{\partial\mathcal{V}_0}{\partial\tau_t^I}\Big|_{\phi_t} + \frac{f(\theta^{t-1})x_t\mathrm{E}_t[\mu_{t+1}(r_t(\theta_t)-\rho)]}{(1+\beta)^{t+1}} \\ & + \sum_{s=0}^{\infty}\frac{\mathrm{E}_0\left[\mu_{s+1}\left((1+\rho-\phi_s)\,\frac{\partial s_s}{\partial\tau_t^I}\Big|_{\phi_t} + \tau_s^I(r_s(\theta_s)-\rho)\,\frac{\partial x_s}{\partial\tau_t^I}\Big|_{\phi_t}\right)\right]}{(1+\beta)^{s+1}} = 0,\end{aligned} \tag{145}$$

$$\begin{aligned}\frac{\partial\mathcal{L}}{\partial\phi_t} = 0: \quad & \frac{\partial\mathcal{V}_0}{\partial\phi_t} - \frac{f(\theta^{t-1})s_t\mathrm{E}_t[\mu_{t+1}]}{(1+\beta)^{t+1}} \\ & + \sum_{s=0}^{\infty}\frac{\mathrm{E}_0\left[\mu_{s+1}\left((1+\rho-\phi_s)\frac{\partial s_s}{\partial\phi_t} + \tau_s^I(r_s(\theta_s)-\rho)\frac{\partial x_s}{\partial\phi_t}\right)\right]}{(1+\beta)^{s+1}} = 0,\end{aligned} \tag{146}$$

$$\frac{\partial\mathcal{L}}{\partial G_{t+1}(\theta_{t+1})} = 0: \quad \frac{\partial\mathcal{V}_0}{\partial G_{t+1}(\theta_{t+1})} - \frac{f(\theta^{t+1})\mu_{t+1}}{(1+\beta)^{t+1}} = 0, \tag{147}$$

$$\frac{\partial\mathcal{L}}{\partial d_{t+1}} = 0: \quad \frac{f(\theta^t)\mu_t}{(1+\beta)^t} - \frac{f(\theta^t)(1+\rho)\mathrm{E}_{t+1}[\mu_{t+1}]}{(1+\beta)^{t+1}} = 0, \tag{148}$$

where the following properties have been used: (i) since public goods are additively separable and lump-sum taxes are fixed, s_s and x_s do not depend on G or d; (ii) since (τ_t^I, ϕ_t) is a bijective reparametrisation of (τ_t^I, τ_t^A), the derivatives with respect to τ_t^I in eq. (145) are taken at a fixed ϕ_t (i.e. jointly with $\mathrm{d}\tau_t^A = -\rho\mathrm{d}\tau_t^I$); the tax rates are set in history θ^{t-1}, so that their mechanical revenue effects carry the density $f(\theta^{t-1})$ and the expectation $\mathrm{E}_t[\cdot]$ over θ_t; (iii) in eq. (147), $G_{t+1}(\theta_{t+1})$ is set in history θ^{t+1}, in which μ_{t+1} is known; (iv) in eq. (148), d_{t+1} is set in history θ^t and appears with coefficient +1 in the period-t constraint (multiplier μ_t, known in

θ^t) and with coefficient $-(1+\rho)$ in the period-$t+1$ constraint (multiplier μ_{t+1}, of which only the conditional expectation $\mathrm{E}_{t+1}[\mu_{t+1}]$ given θ^t is known when d_{t+1} is set).

To simplify the first-order conditions, Lemma (2) is applied: the dynamic Roy identity (22) and the period-t partial derivatives (18), (19) and (20). Eq. (19) is zero by eq. (142), since a marginal increase in the tax on the excess return has no first-order utility cost – holding the after-tax safe return constant. Eq. (18) simplifies to $u'(c_t)s_t/\phi_t$ for the same reason. For the state-contingent public goods, the direct welfare effect is eq. (25) applied at $t+1$: $\partial\mathcal{V}_0/\partial G_{t+1}(\theta_{t+1}) = (1+\beta)^{-(t+1)}\Gamma'(G_{t+1}(\theta_{t+1}))f(\theta^{t+1})$, which carries the density of the history at which the instrument is set, as does the corresponding mechanical revenue effect in eq. (147). Public debt has no direct effect on $\mathcal{V}_t$.

Further, under the particular policy parameterisation, the comparative statics are as follows. The compensated perturbation $\mathrm{d}\tau_t^I$ at fixed ϕ_t leaves the individual's opportunity set unchanged (the after-tax safe return is fixed, and any after-tax exposure remains attainable through ω_t), so that the entire allocation is unchanged and only the portfolio share adjusts:

$$\left.\frac{\partial s_s}{\partial \tau_t^I}\right|_{\phi_t} = 0 \;\forall s, \quad \left.\frac{\partial x_t}{\partial \tau_t^I}\right|_{\phi_t} = \frac{x_t}{1-\tau_t^I}, \quad \left.\frac{\partial x_s}{\partial \tau_t^I}\right|_{\phi_t} = 0 \;\forall s \neq t. \tag{149}$$

This is just a replication of Lemma (3) in the main text. The responses to ϕ_t, $\partial s_s/\partial\phi_t$ and $\partial x_s/\partial\phi_t$, are uncompensated and generally nonzero in all periods s. They never need to be derived explicitly.

The first-order conditions can be simplified by substituting eqs. (18)–(20) and the dynamic Roy identity (22) into eqs. (145)–(148), using the comparative statics (149) in eq. (145) – so that its summation reduces to the Domar-Musgrave term for $s=t$ – and multiplying each condition by $(1+\beta)^{t+1}$. In the mechanical terms, the law of iterated expectations, $\mathrm{E}_t[X] = \mathrm{E}_t[\mathrm{E}_{t+1}[X]]$, gives $\mathrm{E}_t[\mu_{t+1}(r_t(\theta_t)-\rho)] = \mathrm{E}_t[\mathrm{E}_{t+1}[\mu_{t+1}](r_t(\theta_t)-\rho)]$ and $\mathrm{E}_t[\mu_{t+1}] = \mathrm{E}_t[\mathrm{E}_{t+1}[\mu_{t+1}]]$. This gives:

$$\frac{\partial\mathcal{L}}{\partial\tau_t^I} = 0: \quad \frac{f(\theta^{t-1})x_t}{1-\tau_t^I}\mathrm{E}_t\left[\mathrm{E}_{t+1}[\mu_{t+1}](r_t(\theta_t)-\rho)\right] = 0, \tag{150}$$

$$\frac{\partial\mathcal{L}}{\partial\phi_t} = 0: \quad f(\theta^{t-1})\left(\mathrm{E}_t[\mathcal{V}'_{t+1}(\cdot)] - \mathrm{E}_t[\mathrm{E}_{t+1}[\mu_{t+1}]]\right)s_t \tag{151}$$

$$+\sum_{s=0}^{\infty}\frac{\mathrm{E}_0\left[\mu_{s+1}\left((1+\rho-\phi_s)\frac{\partial s_s}{\partial\phi_t} + \tau_s^I(r_s(\theta_s)-\rho)\frac{\partial x_s}{\partial\phi_t}\right)\right]}{(1+\beta)^{s-t}} = 0,$$

$$\frac{\partial\mathcal{L}}{\partial G_{t+1}(\theta_{t+1})} = 0: \quad f(\theta^{t+1})\left(\Gamma'(G_{t+1}(\theta_{t+1})) - \mu_{t+1}\right) = 0, \tag{152}$$

$$\frac{\partial\mathcal{L}}{\partial d_{t+1}} = 0: \quad f(\theta^t)\left(\mu_t - \frac{1+\rho}{1+\beta}\mathrm{E}_{t+1}[\mu_{t+1}]\right) = 0. \tag{153}$$

Eqs. (150)–(153) are the system to be solved. All densities are strictly positive. In eqs. (150), (152) and (153) the density multiplies the entire condition, so that it can be divided out immediately: these conditions hold history by history. In eq. (151) the density only multiplies the direct and mechanical terms, whereas the behavioural sum is an unconditional expectation $\mathrm{E}_0[\cdot]$ that integrates over all histories. This condition can therefore only be divided by the density after the behavioural sum has been shown to vanish. This system is solved in a number

of steps.

First, the multipliers are eliminated. Dividing eq. (152) by the density $f(\theta^{t+1})$ gives $\mu_{t+1} = \Gamma'(G_{t+1}(\theta_{t+1}))$ in every history. Dividing eq. (153) by $f(\theta^t)$ and substituting then gives the government's Euler equation:

$$\Gamma'(G_t(\theta_t)) = \frac{1+\rho}{1+\beta}\mathrm{E}_{t+1}[\Gamma'(G_{t+1}(\theta_{t+1}))], \tag{154}$$

which is the stochastic counterpart of the individual's Euler equation. The government smooths the marginal value of public funds over time at the safe rate. Its implication that is used below is

$$\mathrm{E}_{t+1}[\mu_{t+1}] = \frac{1+\beta}{1+\rho}\Gamma'(G_t(\theta_t)). \tag{155}$$

Hence, the expected marginal value of period-$t+1$ public resources, given θ^t, equals the marginal utility of period-t public goods, corrected for the one-period difference in timing.

Second, the optimal tax on capital income is derived. Dividing eq. (150) by $f(\theta^{t-1})x_t/(1-\tau_t^I) > 0$ and substituting eq. (155):

$$\mathrm{E}_t[\Gamma'(G_t(\theta_t))(r_t(\theta_t)-\rho)] = 0 \quad \Longleftrightarrow \quad \pi_t = -\frac{\mathrm{cov}_t[\Gamma'(G_t(\theta_t)), r_t(\theta_t)]}{\mathrm{E}_t[\Gamma'(G_t(\theta_t))]}, \tag{156}$$

using $\mathrm{E}_t[r_t] - \rho = \pi_t$. At the optimum, the government's risk-adjusted valuation of equity exposure is zero – in exact parallel to the individual's portfolio condition (142) – so that aggregate risk is allocated efficiently between private and public consumption. Eq. (156) pins down the sign of the optimal tax. By the government's Euler equation (154) and budget feasibility, the optimal path of public goods responds positively to the revenue windfall $\tau_t^I(r_t(\theta_t)-\rho)x_t$: public goods provision increases in the return realisation if and only if $\tau_t^I > 0$. With $\Gamma'' < 0$, the covariance in eq. (156) is then negative if and only if $\tau_t^I > 0$. Since $\pi_t > 0$, eq. (156) requires $\tau_t^I > 0$. At $\tau_t^I = 0$, public goods bear no return risk, the covariance is zero, and the left-hand side of eq. (156) equals $\mathrm{E}_t[\Gamma'(G_t(\theta_t))]\pi_t > 0$, so that raising the tax on the excess return strictly raises welfare. The optimal τ_t^I is interior and loads exactly enough aggregate risk onto public consumption that the government's implied risk premium equals the market risk premium. This proves the capital income tax part of Proposition 7.

Third, the optimal after-tax safe return – and thereby the optimal wealth tax – is derived from eq. (151). Consider the candidate solution $\phi_s = 1+\rho$ for all s. At this solution, the intertemporal wedge $1+\rho-\phi_s$ vanishes in every period, so that all saving responses drop out of the summation. The exposure responses drop out as well: since $\partial x_s/\partial\phi_t$ is measurable with respect to θ^{s-1}, the law of iterated expectations gives $\mathrm{E}_0[\mu_{s+1}(r_s(\theta_s)-\rho)\partial x_s/\partial\phi_t] = \mathrm{E}_0[(\partial x_s/\partial\phi_t)\mathrm{E}_s[\mathrm{E}_{s+1}[\mu_{s+1}](r_s(\theta_s)-\rho)]] = 0$ by eq. (150) at date s. Hence *every* behavioural term in eq. (151) vanishes at the candidate solution. The density $f(\theta^{t-1})$ can then be eliminated, and the condition reduces to its direct and mechanical parts. Using $\mathrm{E}_t[\mathcal{V}'_{t+1}] = (1+\beta)u'(c_t)/\phi_t$ from eq. (18) and eq. (155) gives:

$$\frac{(1+\beta)u'(c_t)}{\phi_t}s_t = \frac{1+\beta}{1+\rho}\mathrm{E}_t[\Gamma'(G_t(\theta_t))]s_t \quad \overset{\phi_t=1+\rho}{\Longleftrightarrow} \quad \mathrm{E}_t[\Gamma'(G_t(\theta_t))] = u'(c_t), \tag{157}$$

which is the risk-adjusted Samuelson rule of Proposition 7. At $\phi_t = 1 + \rho$, the first-order condition for ϕ_t is satisfied if and only if public goods provision follows the Samuelson rule in expectation. The configuration $\phi_t = 1+\rho$ for all t, τ_t^I satisfying eq. (156), public goods satisfying eq. (157) and debt satisfying eq. (154) therefore satisfies the complete system (150)–(153). Since $\phi_t \equiv 1+(1-\tau_t^I)\rho-\tau_t^A$, the condition $\phi_t = 1+\rho$ is equivalent to $\tau_t^A = -\tau_t^I\rho$, and, for the ex post wealth tax, $\phi_t \equiv (1-\tau_t^P)(1+(1-\tau_t^I)\rho) = 1+\rho$ is equivalent to $\tau_t^P = -\tau_t^I\rho/(1+(1-\tau_t^I)\rho)$. The optimal wealth tax is negative and exactly undoes the capital-income tax on the safe return. Optimal tax policy is a tax on capital income joint with a rate-of-return allowance. This proves the wealth tax and public goods parts of Proposition 7.

K Proofs Propositions 8, 9 and 10

This appendix derives the optimal time-invariant taxes on capital income and wealth: $\tau_t^I = \tau^I$ and $\tau_t^A = \tau^A$ for all t. Since the safe return ρ is constant, constant tax rates fix the after-tax safe return factor in every period, $\phi_t = \phi \equiv 1 + (1-\tau^I)\rho - \tau^A$ (or $\phi = (1-\tau^P)(1 + (1-\tau^I)\rho)$ for the ex post wealth tax).[39] The bijection between (τ^I, τ^A) and (τ^I, ϕ) therefore applies uniformly, and the government optimises over the constant pair (τ^I, ϕ), while public goods provision, government debt and – where applicable – the state-contingent lump-sum taxes remain unrestricted.

Saving, portfolio shares and exposures are functions of the entire policy path. Since a common tax rate enters the Lagrangian in every period, its first-order condition is the sum over all periods of the per-period derivatives. The behavioural terms in these conditions are therefore *double* sums:

$$\sum_{t=0}^{\infty}\sum_{s=0}^{\infty} \frac{\mu_{s+1}}{(1+\beta)^{s+1}} \left[(1+\rho-\phi)\frac{\partial s_s}{\partial \phi_t} + \tau^I \pi \frac{\partial x_s}{\partial \phi_t}\right], \tag{158}$$

and analogously for τ^I at fixed ϕ.[40]

The three cases of the paper are treated in turn, following the same steps as in the proofs of Propositions 5, 6 and 7: the Lagrangian, the block of first-order conditions, the substitution of Roy's identities and the comparative statics and the solution for each instrument.

K.1 Idiosyncratic risk

Under Assumption 1, idiosyncratic states are i.i.d. over time, so that the risk premium is constant: $\pi_t = \pi$. As in the proof of Proposition 5, all aggregates are deterministic, the individual's portfolio Kuhn-Tucker conditions are eq. (94) with Δ_t satisfying eq. (95), and the government budget constraint is the one entering the Lagrangian (96) with $\phi_t = \phi$ and $\tau_t^I = \tau^I$. Attach deterministic Lagrange multiplier $(1+\beta)^{-(t+1)}\mu_{t+1}$ to the period-$t+1$ budget constraint

[39] A constant ρ is essential: with a time-varying safe return ρ_t, a constant wealth tax cannot compensate a constant capital-income tax in every period, and the reparametrisation in (τ^I, ϕ) would no longer respect the restriction.

[40] Interchanging the order of summation in eq. (158), as is done below when the per-period conditions are collected, requires the double sum to converge absolutely. This is not innocuous: it is precisely the condition that fails at the candidate $\phi = 1 + \rho + \pi$ in Subsection K.1 below.

and deterministic Lagrange multiplier κ to the upper bound $\tau^I \leq 1$. The Lagrangian is then given by:

$$\mathcal{L} = \mathcal{V}_0(a_0) + \sum_{t=0}^{\infty} \frac{\mu_{t+1}}{(1+\beta)^{t+1}} \Big((1+\rho-\phi)s_t + \tau^I \pi x_t + T_t + d_{t+2} - (1+\rho)d_{t+1} - G_{t+1} \Big) + \kappa \left(1 - \tau^I\right). \tag{159}$$

The first-order necessary conditions are:

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial \tau^I} = 0: \quad & \sum_{t=0}^{\infty} \frac{\partial \mathcal{V}_0}{\partial \tau_t^I}\bigg|_{\phi_t} + \sum_{t=0}^{\infty} \frac{\mu_{t+1}\pi x_t}{(1+\beta)^{t+1}} \\
& + \sum_{t=0}^{\infty} \sum_{s=0}^{\infty} \frac{\mu_{s+1}}{(1+\beta)^{s+1}} \left[(1+\rho-\phi) \left. \frac{\partial s_s}{\partial \tau_t^I} \right|_{\phi_t} + \tau^I \pi \left. \frac{\partial x_s}{\partial \tau_t^I} \right|_{\phi_t} \right] - \kappa = 0,
\end{aligned} \tag{160}$$

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial \phi} = 0: \quad & \sum_{t=0}^{\infty} \frac{\partial \mathcal{V}_0}{\partial \phi_t} - \sum_{t=0}^{\infty} \frac{\mu_{t+1} s_t}{(1+\beta)^{t+1}} \\
& + \sum_{t=0}^{\infty} \sum_{s=0}^{\infty} \frac{\mu_{s+1}}{(1+\beta)^{s+1}} \left[(1+\rho-\phi) \frac{\partial s_s}{\partial \phi_t} + \tau^I \pi \frac{\partial x_s}{\partial \phi_t} \right] = 0,
\end{aligned} \tag{161}$$

$$\frac{\partial \mathcal{L}}{\partial G_{t+1}} = 0: \quad \frac{\partial \mathcal{V}_0}{\partial G_{t+1}} - \frac{\mu_{t+1}}{(1+\beta)^{t+1}} = 0, \tag{162}$$

$$\frac{\partial \mathcal{L}}{\partial d_{t+1}} = 0: \quad \frac{\mu_t}{(1+\beta)^t} - \frac{(1+\rho)\mu_{t+1}}{(1+\beta)^{t+1}} = 0, \tag{163}$$

$$\kappa \geq 0, \qquad 1 - \tau^I \geq 0, \qquad \kappa(1 - \tau^I) = 0, \tag{164}$$

where, as in the proof of Proposition 5, the derivatives with respect to τ^I are taken at a fixed ϕ, and s_s and x_s do not depend on G or d. The first-order conditions (162) and (163) for public goods and debt are identical to those in the proof of Proposition 5, since these instruments are unrestricted. The first-order conditions (160) and (161) are the sums over t of the per-period conditions (97) and (98).

Substituting the dynamic Roy identity (22) and the partial derivatives (18), (19) and (20) of Lemma (2) and the comparative statics (102) for periods in which the no-leverage constraint

is slack and (103) for periods in which it binds, gives the simplified system:

$$\frac{\partial \mathcal{L}}{\partial \tau^I} = 0: \quad \sum_{t=0}^{\infty} \frac{x_t(\mu_{t+1}\pi - \Delta_t) + \mu_{t+1}\tau^I \pi \left.\frac{\partial x_t}{\partial \tau_t^I}\right|_{\phi_t}}{(1+\beta)^{t+1}} \tag{165}$$
$$+ \sum_{t=0}^{\infty}\sum_{s=0}^{\infty} \frac{\mu_{s+1}}{(1+\beta)^{s+1}} \left[(1+\rho-\phi) \left.\frac{\partial s_s}{\partial \tau_t^I}\right|_{\phi_t} + \tau^I \pi \left.\frac{\partial x_s}{\partial \tau_t^I}\right|_{\phi_t}\right] = \kappa,$$

$$\frac{\partial \mathcal{L}}{\partial \phi} = 0: \quad \sum_{t=0}^{\infty} \frac{(\mathrm{E}_t[\mathcal{V}'_{t+1}(\cdot)] - \mu_{t+1})s_t}{(1+\beta)^{t+1}} \tag{166}$$
$$+ \sum_{t=0}^{\infty}\sum_{s=0}^{\infty} \frac{\mu_{s+1}}{(1+\beta)^{s+1}} \left[(1+\rho-\phi)\frac{\partial s_s}{\partial \phi_t} + \tau^I \pi \frac{\partial x_s}{\partial \phi_t}\right] = 0,$$

$$\frac{\partial \mathcal{L}}{\partial G_{t+1}} = 0: \quad \Gamma'(G_{t+1}) = \mu_{t+1}, \tag{167}$$

$$\frac{\partial \mathcal{L}}{\partial d_{t+1}} = 0: \quad \mu_t = \frac{(1+\rho)\mu_{t+1}}{(1+\beta)}, \tag{168}$$

together with eq. (164), where in eq. (165) a period-t term has $\Delta_t = 0$ and $\partial x_t/\partial \tau_t^I|_{\phi_t} = x_t/(1-\tau^I)$ if the period-t portfolio is interior, and $\Delta_t \geq 0$ and $\partial x_t/\partial \tau_t^I|_{\phi_t} = \partial s_t/\partial \tau_t^I|_{\phi_t}$ if it is at the corner. For interior periods, the saving responses in the summation vanish by eq. (102).

First, the tax on capital income is derived. Suppose $\tau^I < 1$ at an optimum, so that $\kappa = 0$ by eq. (164). Every period in which the portfolio is interior contributes $\mu_{t+1}\pi x_t/(1-\tau^I) > 0$ to the left-hand side of eq. (165), by the argument of Step 1 in the proof of Proposition 5; every period at the corner contributes $x_t(\mu_{t+1}\pi - \Delta_t) \geq \pi x_t(\mu_{t+1} - \mathrm{E}_t[\mathcal{V}'_{t+1}])$ by eq. (95), which is non-negative whenever the marginal value of public funds weakly exceeds expected private marginal utility, plus wedge-weighted saving responses that vanish when the intertemporal wedges are zero, by the argument of Step 2 in that proof. Hence, under the same conditions as in the proof of Proposition 5, the left-hand side of eq. (165) is strictly positive for any $\tau^I < 1$ at which some portfolio is interior, and non-negative otherwise: raising the common tax on capital income at fixed ϕ raises welfare, and along the compensated path the Domar-Musgrave effect drives every portfolio to the corner. At the optimum, therefore, $\tau^I = 1$ and $\omega_t = 1$ for all t, with the multiplier on the upper bound given by

$$\kappa = \sum_{t=0}^{\infty} \frac{\pi s_t \left[\Gamma'(G_{t+1}) - u'(c_{t+1})\right]}{(1+\beta)^{t+1}} + \mathcal{W} \sum_{t=0}^{\infty}\sum_{s=0}^{\infty} \frac{\Gamma'(G_{s+1})}{(1+\beta)^{s+1}} \left.\frac{\partial s_s}{\partial \tau_t^I}\right|_{\phi_t} \geq 0, \qquad \mathcal{W} \equiv 1 + \rho + \pi - \phi, \tag{169}$$

using $\Delta_t = \pi u'(c_{t+1})$, $x_t = s_t$ and eq. (167) at $\tau^I = 1$: this is the sum over t of the per-period multipliers in eq. (112) at a common wedge $\mathcal{W}$. This proves part 3 of Proposition 8; the constants $\tau^I = 1$ and $\omega_t = 1$ coincide with part 3 of Proposition 5, so the restriction is not binding on this margin.

Second, the intertemporal margin. Impose $\tau^I = 1$ and $x_s = s_s$ in eq. (166). Under full insurance $\mathrm{E}_t[\mathcal{V}'_{t+1}] = u'(c_{t+1})$, and the bracket in the summation equals $\mathcal{W}\,\partial s_s/\partial \phi_t$. Substituting

$\mu_{s+1} = \Gamma'(G_{s+1})$ from eq. (167) yields:

$$\sum_{t=0}^{\infty} \frac{s_t}{(1+\beta)^{t+1}} \left[u'(c_{t+1}) - \Gamma'(G_{t+1})\right] = -\mathcal{W} \sum_{t=0}^{\infty} \sum_{s=0}^{\infty} \frac{\Gamma'(G_{s+1})}{(1+\beta)^{s+1}} \frac{\partial s_s}{\partial \phi_t}, \tag{170}$$

which is the sum over t of the per-period conditions (113) at a common wedge $\mathcal{W}$: with a single after-tax safe return, only the discounted sum of the per-period Ramsey conditions must hold.

Before solving eq. (170), note how the two intertemporal prices evolve. Public goods and public debt are unrestricted, so eqs. (167) and (168) hold period by period and give $\Gamma'(G_{t+1}) = \mu_{t+1} = \mu_1 \lambda^t$ with $\lambda \equiv (1+\beta)/(1+\rho)$. At $\tau^I = 1$ consumption is deterministic, so that the individual's Euler equation (7) gives $u'(c_{t+1}) = u'(c_1)\nu^t$ with $\nu \equiv (1+\beta)/\phi$. Hence:

$$z_t \equiv \frac{\Gamma'(G_{t+1})}{u'(c_{t+1})} = z_0 \left(\frac{\phi}{1+\rho}\right)^t, \qquad z_0 \equiv \frac{\mu_1}{u'(c_1)}. \tag{171}$$

The wedge between the government's saving technology and the private sector's does not cancel: it compounds at the gross rate $\phi/(1+\rho)$. The restriction to time-invariant tax rates replaces the per-period Samuelson rule by the single condition (170), so that a drifting z_t is no longer immediately contradictory – unlike in the proof of Proposition 5, where the per-period Samuelson rule and the two Euler equations cannot hold simultaneously.

Consider first the candidate with a slack government budget constraint, $\mathcal{W} = 0$, i.e. $\phi = 1 + \rho + \pi$ and $\tau^A = -(\rho + \pi)$ (or $\tau^P = -(\rho + \pi)$, since $\phi \equiv 1 - \tau^A = 1 - \tau^P$ at $\tau^I = 1$). The right-hand side of eq. (170) vanishes. Substituting eq. (171) and $u'(c_{t+1}) = u'(c_1)\nu^t$, and using $(1+\beta)^{-t}\nu^t = \phi^{-t}$ and $(1+\beta)^{-t}\lambda^t = (1+\rho)^{-t}$, the condition becomes:

$$\mu_1 \sum_{t=0}^{\infty} \frac{s_t}{(1+\rho)^t} = u'(c_1) \sum_{t=0}^{\infty} \frac{s_t}{(1+\rho+\pi)^t}. \tag{172}$$

The right-hand side is finite by the transversality condition of the individual, which requires saving to grow at a rate below $\phi = 1 + \rho + \pi$. The left-hand side discounts the same tax base at the *government's* rate $1+\rho$, and converges only if saving grows at a rate below $1+\rho$. Whenever it does not, eq. (172) admits only $\mu_1 = 0$: there is no interior optimum and the government's problem is unbounded – the government borrows at $1+\rho$ while the economy transforms resources at $1+\rho+\pi$, so that the implied arbitrage is never exhausted. The candidate $\mathcal{W} = 0$ is therefore not an optimum, exactly as in the proof of Proposition 5.

Consider next $\mathcal{W} > 0$, i.e. $\tau^A = \tau^P > -(\rho + \pi)$. Provided saving does not decrease in the after-tax return ($\partial s_s/\partial \phi_t \geq 0$ for all s and t), the right-hand side of eq. (170) is negative, so that

$$\sum_{t=0}^{\infty} \frac{s_t}{(1+\beta)^{t+1}} \left[\Gamma'(G_{t+1}) - u'(c_{t+1})\right] > 0, \tag{173}$$

which is eq. (42): parts 1 and 2 of Proposition 8. On a net basis capital is then taxed, as in part 4. The compounding in eq. (171) stops if and only if $\phi = 1 + \rho$, i.e. $\tau^A = -\rho$ at $\tau^I = 1$, in which case $\lambda = \nu$, the ratio $z_t = z_0$ is constant, both sums in eq. (170) converge under the individual's transversality condition alone, and eq. (170) delivers $z_0 > 1$: public goods are

under-provided by the same factor in every period, and the intertemporal wedge equals the risk premium, $\mathcal{W} = \pi$. Together with eqs. (167) and (168), which pin down the path of public goods and debt as in the proof of Proposition 5, this completes the proof.

K.2 Aggregate risk with state-contingent lump-sum taxes

The set-up, information structure and notation are those of the proof of Proposition 6: the portfolio condition (124) holds, the budget constraint is eq. (125) with $\phi_t = \phi$ and $\tau_t^I = \tau^I$, and the instruments $T_t(\theta_t)$, $G_{t+1}(\theta_{t+1})$ and d_{t+2} remain unrestricted and history-contingent. Attach stochastic Lagrange multiplier $(1+\beta)^{-(t+1)}\mu_{t+1}$ to the period-$t+1$ budget constraint in each history. The Lagrangian is given by:

$$\begin{aligned}\mathcal{L} \;=\; \mathcal{V}_0(a_0) + \sum_{t=0}^{\infty} \frac{1}{(1+\beta)^{t+1}} \mathrm{E}_0\Big[\mu_{t+1}\Big((1+\rho-\phi)s_t + \tau^I(r_t(\theta_t)-\rho)x_t + T_t(\theta_t) \\ + d_{t+2} - (1+\rho)d_{t+1} - G_{t+1}(\theta_{t+1})\Big)\Big].\end{aligned} \tag{174}$$

The first-order conditions for $T_t(\theta_t)$, $G_{t+1}(\theta_{t+1})$ and d_{t+1} are those of the proof of Proposition 6, eqs. (129), (130) and (131), since these instruments are unrestricted; the first-order conditions for the common tax rates are the sums over t of eqs. (127) and (128):

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial \tau^I} = 0: \quad & \sum_{t=0}^{\infty} \frac{\partial \mathcal{V}_0}{\partial \tau_t^I}\Big|_{\phi_t} + \sum_{t=0}^{\infty} \frac{f(\theta^{t-1}) x_t \mathrm{E}_t[\mu_{t+1}(r_t(\theta_t)-\rho)]}{(1+\beta)^{t+1}} \\ & + \sum_{t=0}^{\infty}\sum_{s=0}^{\infty} \frac{\mathrm{E}_0\left[\mu_{s+1}\left((1+\rho-\phi)\,\frac{\partial s_s}{\partial \tau_t^I}\Big|_{\phi_t} + \tau^I(r_s(\theta_s)-\rho)\,\frac{\partial x_s}{\partial \tau_t^I}\Big|_{\phi_t}\right)\right]}{(1+\beta)^{s+1}} = 0,\end{aligned} \tag{175}$$

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial \phi} = 0: \quad & \sum_{t=0}^{\infty} \frac{\partial \mathcal{V}_0}{\partial \phi_t} - \sum_{t=0}^{\infty} \frac{f(\theta^{t-1}) s_t \mathrm{E}_t[\mu_{t+1}]}{(1+\beta)^{t+1}} \\ & + \sum_{t=0}^{\infty}\sum_{s=0}^{\infty} \frac{\mathrm{E}_0\left[\mu_{s+1}\left((1+\rho-\phi)\frac{\partial s_s}{\partial \phi_t} + \tau^I(r_s(\theta_s)-\rho)\frac{\partial x_s}{\partial \phi_t}\right)\right]}{(1+\beta)^{s+1}} = 0,\end{aligned} \tag{176}$$

where the per-period derivatives $\partial \mathcal{V}_0/\partial \tau_t^I|_{\phi_t}$ and $\partial \mathcal{V}_0/\partial \phi_t$ are taken in the history θ^{t-1} at which the period-t rates would be set, so that each per-period term carries the density $f(\theta^{t-1})$ and the sums over t integrate over these histories.

Substituting the dynamic Roy identity (22), the partial derivatives (18)–(21) of Lemma (2) and the comparative statics (132) – so that the summation in eq. (175) reduces to the Domar-Musgrave terms – gives the simplified system: eqs. (135), (136) and (137) for the unrestricted

instruments, together with

$$\frac{\partial \mathcal{L}}{\partial \tau^I} = 0: \quad \frac{1}{1-\tau^I} \sum_{t=0}^{\infty} \frac{1}{(1+\beta)^{t+1}} \mathrm{E}_0 \left[x_t \, \mathrm{E}_t \left[\mathrm{E}_{t+1}[\mu_{t+1}](r_t(\theta_t) - \rho) \right] \right] = 0, \tag{177}$$

$$\frac{\partial \mathcal{L}}{\partial \phi} = 0: \quad \sum_{t=0}^{\infty} \frac{1}{(1+\beta)^{t+1}} \mathrm{E}_0 \left[\left(\mathrm{E}_t[\mathcal{V}'_{t+1}(\cdot)] - \mathrm{E}_t[\mathrm{E}_{t+1}[\mu_{t+1}]] \right) s_t \right] \tag{178}$$

$$+ \sum_{t=0}^{\infty} \sum_{s=0}^{\infty} \frac{\mathrm{E}_0 \left[\mu_{s+1} \left((1+\rho-\phi) \frac{\partial s_s}{\partial \phi_t} + \tau^I (r_s(\theta_s) - \rho) \frac{\partial x_s}{\partial \phi_t} \right) \right]}{(1+\beta)^{s+1}} = 0,$$

where the densities $f(\theta^{t-1})$ have been absorbed into E_0.

The system is solved by verifying the candidate $\phi = 1 + \rho$, an arbitrary $\tau^I \in [0, 1)$ and lump-sum taxes, public goods and debt satisfying eqs. (135), (136) and (137). First, since these three conditions are unchanged, the derivations of the proof of Proposition 6 carry over verbatim: eqs. (136) and (137) give the government's Euler equation (138) and its implication (139); at the candidate the intertemporal wedge is zero and eq. (135) yields the Samuelson rule state by state, eq. (141); and eq. (141) makes every exposure response priced at zero, eq. (140), for every s. Second, the tax on capital income: by eq. (140), every term of the sum in eq. (177) is identically zero at the candidate, so the condition holds for every $\tau^I \in [0, 1)$ – the common tax on capital income is indeterminate, exactly as in part 3 of Proposition 6, and the endpoint $\tau^I = 1$ is excluded for the same reason. Third, the after-tax safe return: at the candidate every behavioural term in eq. (178) vanishes (zero wedge, zero-priced exposures), and each term of the first sum vanishes as well, since eq. (135) gives $\mathrm{E}_t[\mathrm{E}_{t+1}[\mu_{t+1}]] = \mathrm{E}_t[u'(c_{t+1})] = \mathrm{E}_t[\mathcal{V}'_{t+1}]$; any $\phi \neq 1 + \rho$ would distort the intertemporal margin while a non-distorting instrument is available and is dominated, so $\phi = 1 + \rho$, i.e. $\tau^A = -\tau^I \rho$ (or $\tau^P = -\tau^I \rho / (1 + (1 - \tau^I)\rho)$). The candidate attains the first best by the argument of the proof of Proposition 6, and its tax rates are constant, so the restriction is not binding. This proves Proposition 9.

K.3 Aggregate risk with state-contingent public goods

The set-up, information structure and notation are those of the proof of Proposition 7: the lump-sum taxes are fixed, the portfolio condition (142) holds, the budget constraint is eq. (143) with $\phi_t = \phi$ and $\tau^I_t = \tau^I$, and $G_{t+1}(\theta_{t+1})$ and d_{t+2} remain unrestricted. The Lagrangian is eq. (174) with $T_t(\theta_t)$ fixed. The first-order conditions for $G_{t+1}(\theta_{t+1})$ and d_{t+1} are eqs. (147) and (148); the first-order conditions for the common tax rates are eqs. (175) and (176), now with the responses to fixed lump-sum taxes absent. Substituting the dynamic Roy identity (22), the partial derivatives (18)–(20) of Lemma (2) and the comparative statics (149) gives the simplified

system: eqs. (152) and (153) together with

$$\frac{\partial \mathcal{L}}{\partial \tau^I} = 0: \quad \frac{1}{1-\tau^I}\sum_{t=0}^{\infty}\frac{1}{(1+\beta)^{t+1}}\mathrm{E}_0\left[x_t\,\mathrm{E}_t\left[\mathrm{E}_{t+1}[\mu_{t+1}](r_t(\theta_t)-\rho)\right]\right]=0, \tag{179}$$

$$\frac{\partial \mathcal{L}}{\partial \phi} = 0: \quad \sum_{t=0}^{\infty}\frac{1}{(1+\beta)^{t+1}}\mathrm{E}_0\left[\left(\mathrm{E}_t[\mathcal{V}'_{t+1}(\cdot)]-\mathrm{E}_t[\mathrm{E}_{t+1}[\mu_{t+1}]]\right)s_t\right] \tag{180}$$

$$+\sum_{t=0}^{\infty}\sum_{s=0}^{\infty}\frac{\mathrm{E}_0\left[\mu_{s+1}\left((1+\rho-\phi)\frac{\partial s_s}{\partial \phi_t}+\tau^I(r_s(\theta_s)-\rho)\frac{\partial x_s}{\partial \phi_t}\right)\right]}{(1+\beta)^{s+1}}=0.$$

First, the multipliers are eliminated as in the proof of Proposition 7: eqs. (152) and (153) give the government's Euler equation (154) and its implication (155), $\mathrm{E}_{t+1}[\mu_{t+1}] = \frac{1+\beta}{1+\rho}\Gamma'(G_t(\theta_t))$.

Second, the tax on capital income is derived. Substituting eq. (155) into eq. (179) and dropping the positive constants $(1-\tau^I)^{-1}$ and $(1+\rho)^{-1}$ gives the risk-allocation condition in discounted-average form:

$$\sum_{t=0}^{\infty}\frac{1}{(1+\beta)^t}\mathrm{E}_0\left[x_t\,\mathrm{E}_t[\Gamma'(G_t(\theta_t))(r_t(\theta_t)-\rho)]\right]=0, \tag{181}$$

which is eq. (46) of Proposition 10: a single common tax rate can only set the discounted sum of the per-period risk-allocation conditions (156) to zero. Its sign follows as in the proof of Proposition 7: at $\tau^I = 0$, public goods bear no return risk in any period, so that every term in eq. (181) equals $\mathrm{E}_0[x_t\mathrm{E}_t[\Gamma'(G_t(\theta_t))]]\pi_t > 0$, and raising the common tax on the excess return strictly raises welfare; with $\Gamma'' < 0$ and public goods increasing in the return realisation for $\tau^I > 0$, the covariance terms turn negative as τ^I rises, so the optimal τ^I is positive and interior. This proves part 3 of Proposition 10; since the safe return is undistorted at the candidate below, capital is taxed on average in every period, part 4.

Third, the after-tax safe return. Consider the candidate $\phi = 1+\rho$. The intertemporal wedge vanishes, so all saving responses drop out of eq. (180). The exposure responses are, by the law of iterated expectations, $\tau^I\sum_t\sum_s(1+\beta)^{-(s+1)}\mathrm{E}_0[(\partial x_s/\partial\phi_t)\mathrm{E}_s[\mathrm{E}_{s+1}[\mu_{s+1}](r_s(\theta_s)-\rho)]]$, which vanish if and only if the per-period risk-allocation conditions $\mathrm{E}_s[\Gamma'(G_s(\theta_s))(r_s(\theta_s)-\rho)]=0$ hold for all s – which eq. (181) guarantees only on average. If they hold – as is the case whenever the unrestricted optimum of Proposition 7 is itself time-invariant, since it then satisfies the restricted first-order conditions and is the restricted optimum – every behavioural term in eq. (180) vanishes at the candidate, and using $\mathrm{E}_t[\mathcal{V}'_{t+1}] = (1+\beta)u'(c_t)/\phi$ from eq. (18) and eq. (155) at $\phi = 1+\rho$ the condition reduces to

$$\sum_{t=0}^{\infty}\frac{1}{(1+\beta)^t}\mathrm{E}_0\left[\left(\mathrm{E}_t[\Gamma'(G_t(\theta_t))]-u'(c_t)\right)s_t\right]=0, \tag{182}$$

which is eq. (44): the risk-adjusted Samuelson rule holds in discounted-average form, and $\phi = 1+\rho$ is equivalent to $\tau^A = -\tau^I\rho$ (or $\tau^P = -\tau^I\rho/(1+(1-\tau^I)\rho)$). This proves parts 1 and 2 of Proposition 10. If the per-period conditions do not all hold, the exposure terms in eq. (180) do not vanish at $\phi = 1+\rho$, and the optimal ϕ solves eq. (180) in full – which, after substituting

eqs. (18), (155) and $\mu_{s+1} = \Gamma'(G_{s+1}(\theta_{s+1}))$, is eq. (45) – so that generally $\phi \neq 1 + \rho$. This proves the second statement of part 2 of Proposition 10 and completes the proof.

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